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HOW LIGHTNING TORTUOSITY AFFECTS THE ELECTROMAGNETIC FIELDS BY AUGMENTING THEIR EFFECTIVE DISTANCE

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Abstract—A novel approach for developing the electromagnetic fields from a lightning return stroke which follows a tortuous path will be presented. The proposed model is unique in that it recognizes that the symmetrical tortuosity of lightning directly impacts the observable distance r, which in turn, alters the resulting electromagnetic fields. In the literature, lightning return stroke models typically employ the assumption that the cloud-to-ground path is straight. Although this assumption yields fairly consistent results across an array of varying approaches, it does not account for lightning's natural physical appearance. Furthermore, straight-line models only account for the cloud-to-ground discharges and do not address branching and/or cloud-to-cloud discharges which are far more common. In reality, the "steps" which make up the lightning channel's initial descent are staggered or tortuous with respect to each other. Given this fact, the upward traveling current wavefront which follows this prescribed path will exhibit the same characteristics. In doing so, each current segment,

which forms along its respective step, induces electromagnetic fields with angular aggregates that propagate outward from their origin. This, in turn, will generate spatial points where there are fields of higher and lower intensities. The results presented in this paper will show how the effective observable distance due to symmetrical tortuosity alters the resulting electromagnetic fields. Furthermore, it will be shown that as the observable distance r is increased, results from the proposed model closely resemble the straight-line model which strongly suggests that symmetrical tortuosity is only influential at relatively close distances.

1. INTRODUCTION

Lightning poses a major problem to the world's technological infrastructure due to the industry's reliance upon electronics which are extremely susceptible to both direct and indirect effect strikes. Direct lightning strikes can cause considerable damage upon striking an object given the tremendous amount of current they carry. Some of the entities commonly affected include: personal electronics, power supply generators, commercial buildings, and residential structures. In fact, lightning causes several billion dollars in damage each year in the U.S. and is one of the most prolific causes of forest fires. In addition to their economic effects, direct lightning strikes are one of the leading causes of weather-related deaths in the United States. Florida comes out on the top when it comes to lightning-related fatalities and is considered the lightning capital of the U.S. This is largely due to the collision of the west and east coast sea breezes which create thunderstorms over the peninsula primarily during the summer months.

Lightning generates additional and even more elusive constituents that can wreak havoc upon modern electronics. These indirect effects from lightning strikes do not pose much of a physical hazard to people but can cause considerable damage to sensitive electronic components. In principle, the return stroke generates electric and magnetic fields which propagate outward from their point of impact. As these fields come in contact with electronic components they couple into their conductive hardware. Once this occurs, secondary voltage and current sources are generated which subsequently add to those already present. This can lead to catastrophic and/or latent failures to the components or systems that they affect.

Given the impact of lightning strikes posing to life and property, considerable research has been done in order to establish a better understanding of how they develop and identify all of their effects. The

four classes of models defined by Rakov and Uman [1] address the latter to determine and categorize the electromagnetic constituents induced by a lightning strike's return stroke. These electromagnetic fields are, in turn, used to model the types of currents and voltages which may be induced in electrically susceptible entities within an appreciable distance from a lightning strike. For this study, the methods of solving for the electromagnetic fields will utilize a class of "engineering" models in their analysis. These "engineering" models can be separated into several sub-classes, two of which include: the transmission-line (TL) and Traveling-Current-Source (TCS) models [1]. From the two, the TL model was the adopted choice when solving for the electromagnetic fields which result from a tortuous return stroke path.

2. LITERARY BACKGROUND

In reality the path that a lightning's return stroke follows is not straight. It can be thought as numerous adjoining steps at arbitrary angles that originate from a given surface and propagate upward until the lightning channel has become fully discharged. Its appearance has been described by Lichtenberg figures [2] which have a fractal pattern closely resembling the random growth of tree branches. In a previous paper [3], a lightning return stroke was modeled as a straight line in the presence of a perfect conductor. Although this approach should be regarded as a good approximation, it does not represent a geometrically realistic model. In reality, the return stroke is made up of numerous "steps" which are staggered or tortuous on a scale of 1 m to over a 1 km with a mean absolute value of channel azimuth of approximately 16° [4].

Le Vine and Meneghini [5] investigated an arbitrary current filament located above a perfectly conductive ground plane driven by a traveling wave. In their theoretical analysis, they were able to develop exact and approximated solutions for the resulting fields. Their exact solution predicted both radiation and near fields regardless of the distance between the observation point and filament. Whereas, their approximated solution utilized far-field approximation and as a result, only yields electric field radiation constituents which occupy the Fraunhofer region of the filament. However, upon comparing the two solutions, they found the approximate solution failed at the lower frequencies which are of particular importance in lightning studies. Moini et al. [6] utilized an antenna theory (AT) model to predict the electromagnetic fields at several points from a lightning channel at an inclined angle. Their paper presented results which confirm that channel inclination considerably affects the electromagnetic fields at

close distances. Lupo et al. [7], in their analysis, treated each tortuous path as a single line radiator with an arbitrary slope and height. They were able to develop closed-form solutions for the electromagnetic fields by omitting any mathematical approximations. Later, Lupo et al. [8] extended their studies to address the effects branching having on the corresponding fields. Chia and Liew [9] found that if the tortuous path taken by a lightning return stroke was fairly similar to a straight line, the corresponding electric and magnetic fields were quite similar. However, as the results will show, if the path taken by the return stroke becomes more tortuous the findings from Chia and Liew [9] are no longer valid. This consequence stems from the fact that as time varying current elements begin to follow a more staggered path, each current segment will respond by altering its orientation, thus generating electric and magnetic fields that respond accordingly. This response will give rise to electric and magnetic fields which are spatially nonlinear with areas of low field intensities [7] as well as high field intensities.

3. EFFECTIVE DISTANCE

In this paper, we investigate how channel tortuosity directly impacts the distance from which the fields are evaluated. This varying distance is due in part to the arbitrary orientation each current segment makes with respect to an observation point P. The lightning return stroke, as illustrated in Fig. 1, is made up of numerous steps which originate within the base of thunderstorms and extend downward a height H during the initial stages of atmospheric breakdown.

One can describe the length for each step h with the following formula derived by Rubinstein and Uman [3],

$$h = \beta \frac{(ct - \beta z) \pm \sqrt{(\beta ct - z)^2 + r^2 (1 - \beta^2)}}{1 - \beta^2} \tag{1}$$

with the quantity $\beta = v/c$ being the ratio of the current propagation speed along the lightning channel to the speed of light and the variable z represents the position along the vertical axis.

In principle, step leaders are negatively charged and tend to branch off, taking multiple paths as they extend downward towards the Earth. Electric fields, which form during the separation of charges between the Earth's surface and cloud base, force step leaders to propagate in a series of quantized steps. These steps may vary in length between 30 to 50 meters with $1\,\mu s$ durations. Their movement towards the earth can take on paths which tend to be symmetric and
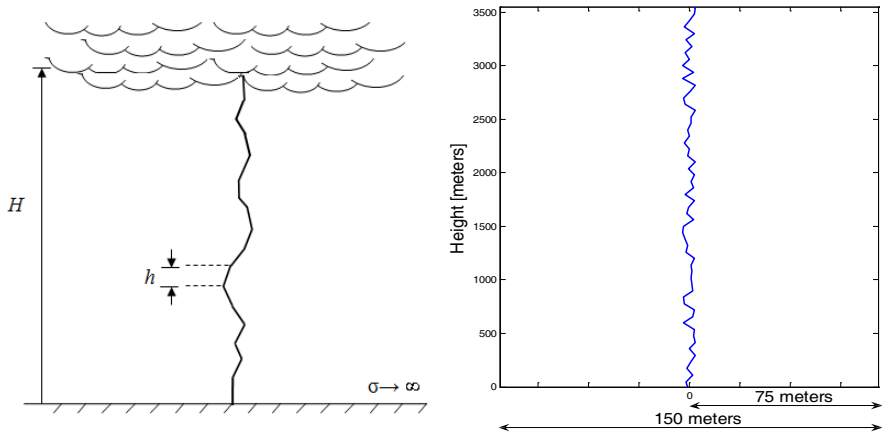
Figure 1. Illustration of a lightning return stroke (left) with the ground-to-cloud base height H and step length h. An example of a return stroke generated using the proposed model (right).

asymmetric in nature. Once an ionized luminescent step leader meets up with an oppositely charged streamer, current segments will form and travel upward along their prescribed path, thereby forming the lightning wavefront.

The method being presented only considers the effect from symmetrical tortuosity, which implies the lightning channel is symmetrical twisting around a vertical channel reference. Doing so allowed the tortuous channel to resemble its straight-line counterpart with the exception of arbitrary kinks along its path. By leveraging this methodology, this study treated the lightning return stroke as a continuous piecewise linear channel whose steps were evaluated independently and only varied in terms of r at given points along the z-axis. Furthermore, the spatial position from which each step originated only retained its predecessor's vertical endpoint which allowed the channel's footprint along the r-axis to remain relatively narrow. The electromagnetic fields which transpired were a culmination of the fields induced by each current segment along the z-axis, an approach similar to the one used by Moini et al. [6]. However, the results presented assume that each current segment's orientation is arbitrary and influences the geometry along the entire channel. Whereas, the findings from [6], focused on how the fields were affected when the entire channel was inclined at a fixed angle. Given the channel is made up of numerous current segments each of which may have its own inclined angle, warrants the investigation this study presents.
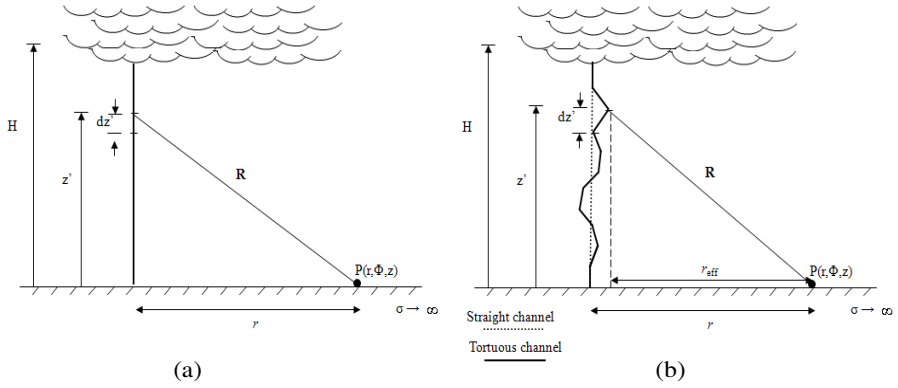
Figure 2. Comparison between the straight-line model and tortuous return stroke model. The variable z' represents the height along the channel at some specified time t.

When the typical straight-line model is considered, the observable distance r remains constant between the lightning channel and observation point P. However, as the channel becomes tortuous, one can no longer make this assumption. An illustration of the two cases is depicted in Fig. 2.

The magnitude R shown in Fig. 2 is described by the following,

$$R = \sqrt{(z - z')^2 + r^2} \tag{2}$$

where r is the observable distance and z is the height along the channel where the fields are evaluated.

From Fig. 2(a), it is apparent that distance r remains fixed as the current wavefront moves upward along the straight channel. However, according to Fig. 2(b), as each current segment orients itself towards the observation point P, the distance between its apex, perpendicular with respect to ground, and the observation point will decrease when compared to the straight-line model. Conversely, an orientation away from point P will have the opposite effect by increasing the distance, again with respect to the straight-line model. This is due to the dependence the observable distance r has on the angle of departure which exists between each current segment and its vertical reference. An important item to note is Fig. 2(b) illustrates the effective distance for a single step located near the top of the channel. Given the lightning channel is made up of numerous adjoining steps at arbitrary angles, it becomes necessary to evaluate r_{eff} for every step within the channel from $z' = 0$ to $z' = H$. A closer examination of the first two steps from Fig. 2(b) is provided by Fig. 3.
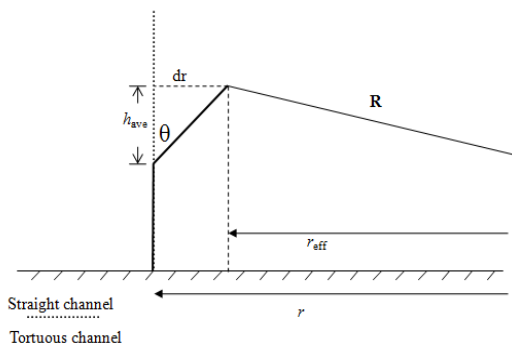
Figure 3. This illustration is a depiction of the first two current segments from Fig. 2(b). The variable dr, is the length between the current segment's apex and its vertical reference. This term is subtracted from the observable distance r in order to correlate the current segment's departure angle with a scalar distance to the observation point P. The angle θ, is the angular departure which occurs within a tortuous channel, and h_{ave} is the average length of each current segment.

As Fig. 3 details, the orientation each current segment makes with respect to the observation point P, directly affects the observable distance r. Consequently, it becomes necessary to account for this change by formulating an expression to describe this new distance. In doing so, we can quantify a relationship between the two distances with the following,

$$r_{eff} = r - h_{ave} \cdot \tan(\theta) \tag{3}$$

where r_{eff} is the effective distance, h_{ave} is the average length of each current segment, and θ is the angle of departure which is bounded by angles greater than $-\pi/2$ and less than $\pi/2$. Fundamentally, the effective distance represents the scalar length between each current segment's apex, perpendicular with respect to ground, and a fixed observation point. Its usage is necessary to account for the angular variability which occurs naturally within a tortuous return stroke. This variability is deterministic to how much the electric and magnetic fields associated with each current segment are affected. Additionally, if we assume the departure angle between each current segment, with respect to its vertical reference is zero, it becomes apparent that the effective distance r_{eff}, and observable distance r, from (3) are equivalent. Thus, the two distances can be used interchangeable to differentiate between straight-line and tortuous return stroke models.

Given a relationship between the observable and effect distances has been established, we can now quickly derive the general expressions used to describe the electromagnetic fields for the effective distance. From Maxwell's equations, we can write the electric and magnetic fields in terms of the vector potential A such that,

$$\mathbf{E}\left(\mathbf{r}_s, t\right) = c^2 \int\limits_{0^-}^{t} \nabla \left(\nabla \cdot \mathbf{A}\right) d\tau - \frac{\partial \mathbf{A}}{\partial t} \tag{4}$$

and

$$\mu_0 \mathbf{H} = \nabla \times \mathbf{A}. \tag{5}$$

Equations (4) and (5) can be used along with the vector potential A,

$$d\mathbf{A}\left(\mathbf{r}_s, t\right) = \frac{\mu_0}{4\pi} \frac{i\left(\mathbf{r}_s, t - R/c\right) dz'}{R} a_z \tag{6}$$

and current distribution,

$$i\left(z', t\right) = I_0 u\left(t - |z'|/v\right) \tag{7}$$

to develop the expressions used to describe the electromagnetic fields from a lightning return stroke. Therefore, upon substituting (6), (7) into (4), (5) one would obtain the general expressions for the electric and magnetic fields in cylindrical coordinates at any point in (r, z) space such that [3],

$$d\mathbf{E}_z = \frac{dz'}{4\pi\varepsilon} \left[\frac{2\left(z - z'\right)^2 - r^2}{R^5} \int\limits_{0^-}^{t} i\left(t - R/c\right) d\tau + \frac{2(z - z')^2 - r^2}{cR^4} i(t - R/c) \right.$$
$$\left. - \frac{r^2}{c^2 R^3} \frac{\partial i\left(t - R/c\right)}{\partial t} \right] \tag{8}$$

and

$$dH_\Phi = \frac{dz'}{4\pi} \left[\frac{r}{cR^2} \frac{\partial\left(t - R/c\right)}{\partial t} + \frac{r}{R^3} i\left(t - R/c\right) \right]. \tag{9}$$

Upon integrating Equations (8), (9) along the $z = 0$ plane from $-h$ to h, and then substituting (3) in place of r, one would obtain the

following,

$$E_z \left(r - h_{ave} \cdot \tan(\theta), 0, t\right)$$

$$= \frac{I_0}{2\pi\varepsilon_0} \left[\frac{-th + \dfrac{2h^2}{v} + \dfrac{(r - h_{ave} \cdot \tan(\theta))^2}{v}}{\left(h^2 + (r - h_{ave} \cdot \tan(\theta))^2\right)^{3/2}} - \frac{1}{(r - h_{ave} \cdot \tan(\theta)) \, v} \right.$$

$$\left. - \frac{(r - h_{ave} \cdot \tan(\theta))^2}{c^2 \left(h^2 + (r - h_{ave} \cdot \tan(\theta))^2\right)^{3/2} \left(\dfrac{1}{v} + \dfrac{h}{c\sqrt{h^2 + (r - h_{ave} \cdot \tan(\theta))^2}}\right)} \right] \quad (10)$$

and

$$H_\Phi \left(r - h_{ave} \cdot \tan(\theta), 0, t\right)$$

$$= \frac{I_0}{2\pi} \left[\frac{h}{(r - h_{ave} \cdot \tan(\theta)) \sqrt{h^2 + (r - h_{ave} \cdot \tan(\theta))^2}} \right.$$

$$\left. + \frac{r - h_{ave} \cdot \tan(\theta)}{\dfrac{c}{v} \left(h^2 + (r - h_{ave} \cdot \tan(\theta))^2\right) + h\sqrt{h^2 + (r - h_{ave} \cdot \tan(\theta))^2}} \right]. \quad (11)$$

where I_0 is the current amplitude, h is the length of each current segment as provided by [3], v is the wavefront speed, c is the speed of light, r is the observable distance, h_{ave} is the average segment length, and θ is the departure angle. These fields (10), (11) correspond to those derived by Rubinstein and Uman [3], with the exception of the observable distance r which has been replaced by the effective distance r_{eff} as provided by (3).

4. SIMULATION OF RESULTS

The following illustrations utilized (10), (11) in the time domain from 0 to 3×10^{-5} in increments of 4×10^{-7} seconds. The angle θ was calculated numerically with the aid of random number generating functions. These functions were used to create the angular variability over the defined intervals described by each plot. By taking the tangent of these randomly generated angles and multiplying it by the average length h_{ave} as shown in (3), one can effectively simulate a channel with symmetrical tortuosity.

As Figs. 4–7 illustrate, the channel tortuosity has a noticeable effect on the electromagnetic fields. This implies that points of lower and higher field intensities can be directly attributed to the variability of the effective distance, r_{eff}.
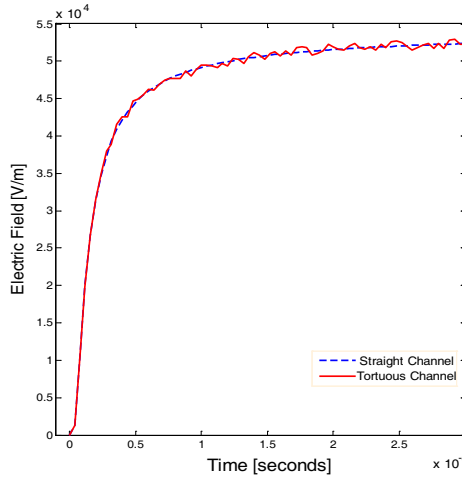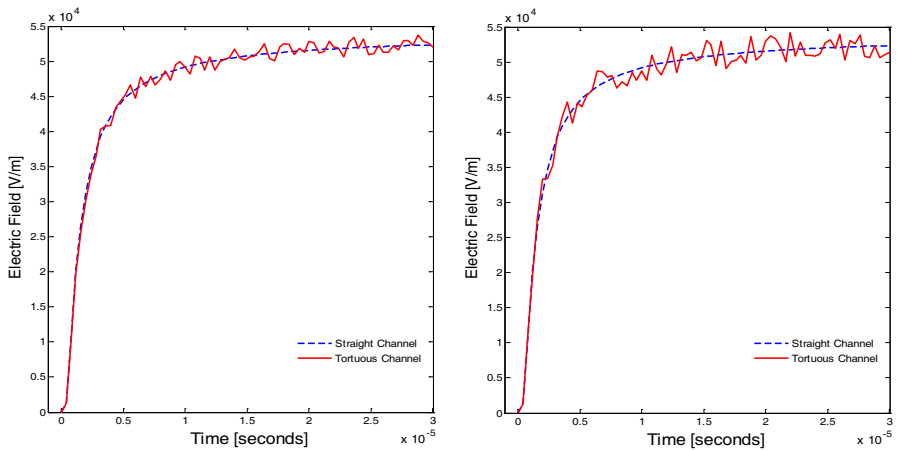

Figure 4. Compares the electric fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ± 10 degrees when $r = 100$ m.

Figure 5. Compares the electric fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ± 20 (left) and ± 30 (right) degrees when $r = 100$ m.

Figure 6. Compares the magnetic fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ±10 degrees when $r = 100$ m.

Figure 7. Compares the magnetic fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ±20 (left) and ±30 (right) degrees when $r = 100$ m.
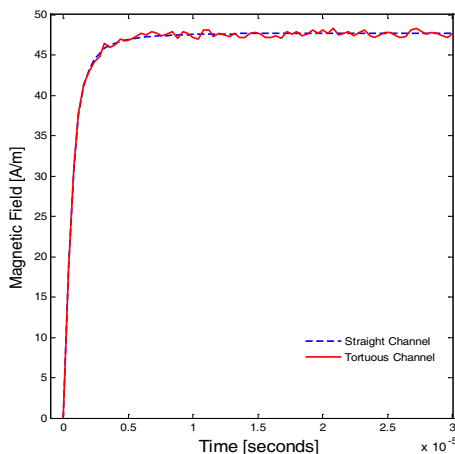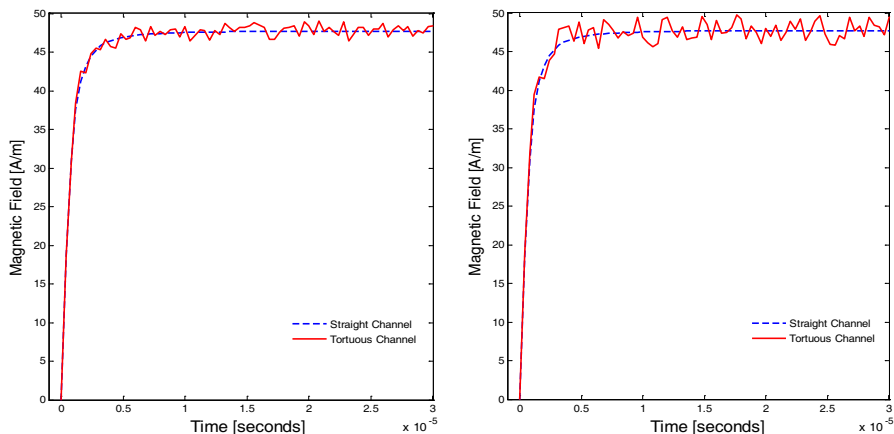
Figures 8–11 highlight an important contribution made by the effective distance. Mainly, symmetrically tortuosity is only effective at relatively close distances. That is, as a return stroke strikes further away from the observation point P, the channel tortuosity plays less of a role in augmenting the electromagnetic fields intensity. Subsequently,

once the observable distance r approaches 1000 meters, the effects from symmetrical tortuosity becomes negligible. Consequently, the electric and magnetic fields distribution become indistinguishable from those generated by a straight-line model.
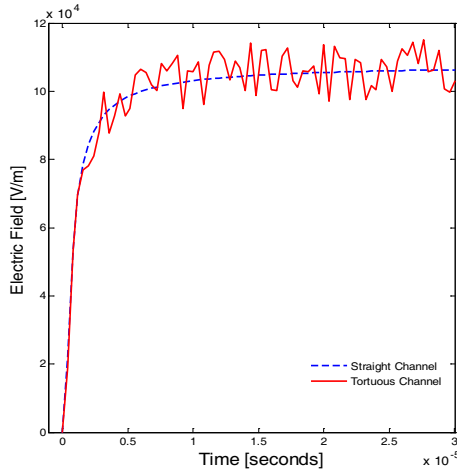

Figure 8. Compares the electric fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ± 30 degrees when $r = 50\,\mathrm{m}$.
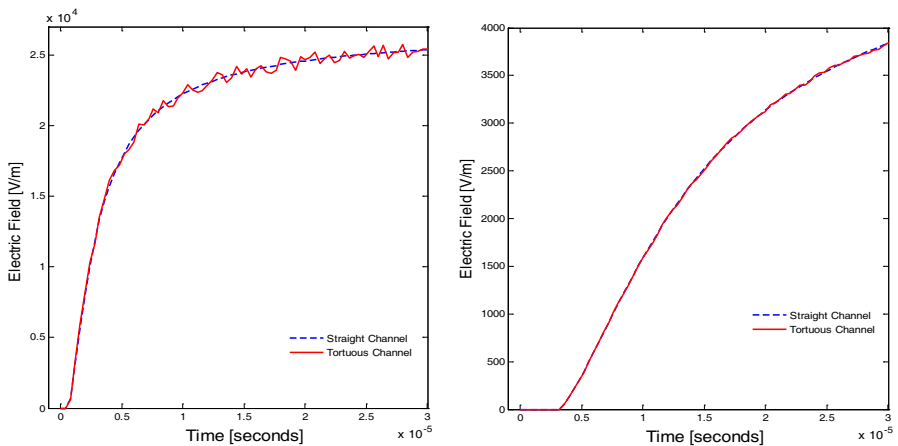

Figure 9. Compares the electric fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ± 30 degrees when $r = 200\,\mathrm{m}$ (left) and $r = 1000\,\mathrm{m}$ (right).
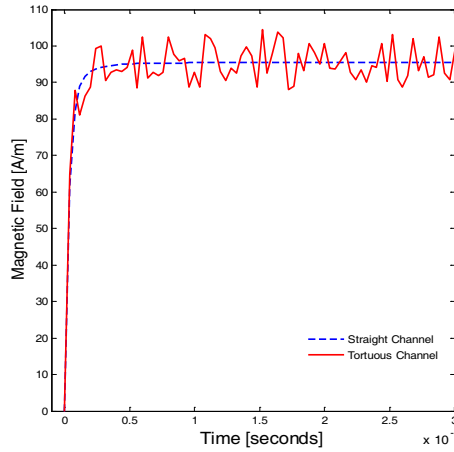
Figure 10. Compares the magnetic fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ± 30 degrees when $r = 50\,\text{m}$.
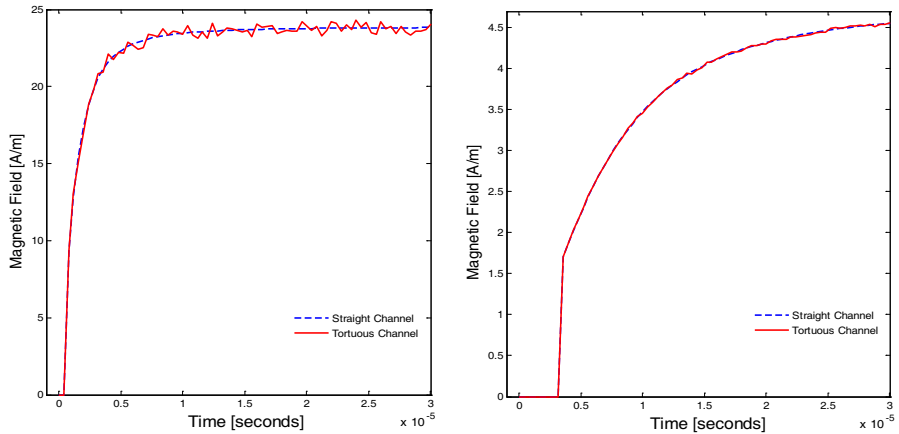

Figure 11. Compares the magnetic fields induced by a straight and tortuous return stroke with adjoining current segments which vary over the interval ± 30 degrees when $r = 200\,\text{m}$ (left) and $r = 1000\,\text{m}$ (right).

5. CONCLUSION

A novel approach for describing a return stroke which follows a tortuous path has been presented. The proposed model is unique in that it recognizes that the symmetrical tortuosity of lightning

directly impacts the observable distance r, which in turn, alters the resulting electromagnetic fields. In doing so, an effective distance needs to be introduced, which accounts for the variability of r due to channel tortuosity. The results presented highlight two important contributions made by the introduction of the effective distance. First, the degree of channel tortuosity determines the effective distance which directly impacts the electromagnetic fields by creating points of lower and higher field intensities. Second, as the observable distance r becomes approximately three orders of magnitude greater than the effective distance r_{eff}, the contribution from channel tortuosity becomes negligible. This strongly suggests that symmetrical tortuosity is only influential at relatively close distances, and the electromagnetic fields distribution becomes indistinguishable from straight-line models for distances greater than 1000 meters.

ACKNOWLEDGMENT

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