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# Surface Area and Volume [10th grade]

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# UNDERSTANDING BY DESIGN

## Unit Cover Page

**Unit Title: Surface Area and Volume**

**Grade Level: 10**

**Subject/Topic Area(s): Geometry**

**Designed By: Maeve Goetz**

**Time Frame: 3 weeks; Block Schedule**

**Brief Summary of Unit**

The students will be introduced to the surface area and volume of cylinders, prisms, pyramids, and cones. The main questions on which the unit is focused are:

- What are different ways that three-dimensional objects are described or depicted?
- What types of problems are solved using three-dimensional objects?

They will participate in activities to help them create the formulas for both surface area and volume and become comfortable using these formulas to solve real-world problems and create their own applications. They will also analyze the relationship between surface area and volume.

The culminating activity in this unit is a project in which the students will design a new container for M&M's. They will compare and contrast different shapes and sizes in order to determine the best container. They will calculate the surface areas and volumes of their choices and pick one container to construct. They will draw a net of this figure with all the specifications needed to manufacture the container and draw different views of the container. Finally, they will present their finished product to the class and reflect on the process.

# Unit: Surface Area & Volume
# Grade: 10

## Stage 1: Desired Results

### Understandings

*Students will understand that:*
*- There are many different ways to describe & represent three-dimensional figures.*
*- There are many useful applications of surface area and volume in the real-world.*

### Essential Questions

- What are different ways that three-dimensional objects are described or depicted?
- What types of problems are solved using three-dimensional objects?

### Knowledge & Skill

§111.34.Geometry
(d) Dimensionality and the geometry of location: knowledge and skills and performance descriptions.
(1) The student analyzes the relationship between three-dimensional objects and related two-dimensional representations and uses these representations to solve problems. Following are performance descriptions.
(A) The student describes, and draws cross sections and other slices of three-dimensional objects.
(B) The student uses nets to represent and construct three-dimensional objects.
(C) The student uses top, front, side, and corner views of three-dimensional objects to create accurate and complete representations and solve problems.

(e) Congruence and the geometry of size: knowledge and skills and performance descriptions.
(1) The student extends measurement concepts to find area, perimeter, and volume in problem situations. Following are performance descriptions.
(A) The student finds areas of regular polygons and composite figures.
(D) The student finds surface areas and volumes of prisms, pyramids, spheres, cones, and cylinders in problem situations.

## Stage 2: Assessment Evidence

*Performance Task:*
*See attached description and rubric.*

Other evidence:

*Writing assignment comparing the surface area and the volume of a three-dimensional figure.*
*Various Homework Assignments.*
*Quiz covering surface area.*
*Quiz covering volume.*
*Cumulative Test covering surface area and volume.*

## Stage 3: Learning Activities

**Day One:**
Objectives: Students will be able to:
- dissect a three-dimensional figure into its components faces.
- represent a three-dimensional figure's surface area as the sum of the areas of its component faces.
- draw the net of a three-dimensional figure in many ways.
- explain some of the importance of surface area.

Warm-up: Have the students find the area and perimeter/circumference for the following: a triangle, rectangle, a regular polygon (with more than 4 sides), and a circle.

Go over the warm-up, asking for student volunteers to explain the solution to one of the problems to the class.

Give the students the project description and rubric. Go over each, give the deadline, and answer any questions.

Introduce the idea of surface area. What is it and why is knowing it important? Give the students a situation like: if you want to make a doghouse for your new puppy how would you begin? They need to come to the conclusion at some stage that they need to figure out how much material they need in order to build it (figure out the surface area).

Once students have a good idea of what surface area is and how it can be important, explain the difference between lateral and total surface area. To help the students remember lateral surface area, ask them what it means if you were to move laterally, what would it look like? If you made a lateral pass in football, in what direction are you making the pass? Now that they have an idea of surface area, explain the activity for the day and divide the class into partners.

Each pair will receive each of the following: a rectangular prism, a triangular prism, a rectangular pyramid, a triangular pyramid, a cylinder, and a pentagonal prism. These figures should be constructed from a net and have each edge attached to another with tape. In this way, the students can dissect the 3-D figure into many different nets and have an easier time drawing them. The students will also add up the individual areas of the faces with and without the area(s) of the base(s) in order to find the total and lateral surface areas, respectively.
**Homework: Students will complete a set of problems finding both the lateral and total surface areas of various cylinders, prisms, and pyramids. The nets need to be given to them on the paper, along with a drawing of how the three-dimensional figures would look.**

**Day Two:**

Objectives: Students will be able to:

- represent a three-dimensional figure's surface area as the sum of the areas of its component faces.
- Compare and contrast the characteristics of cylinders, prisms, & pyramids.
- <u>draw the net of a three-dimensional figure in many ways.</u>
- derive the formulas for surface area.
- <u>explain some of the importance of surface area.</u>

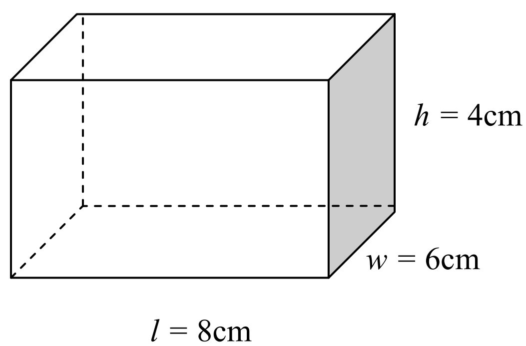
Warm-up: What problems did you have with the homework? What similarities and differences are there between the different three-dimensional figures you studied?

While the students are answering the warm-up questions, check the homework, grading on effort (if the students have shown their work and tried to solve the problems or have written questions or explanations when they got stuck).

Make a list of problems the students had with the homework and address those questions as the homework is reviewed. Allow students to volunteer to present a problem and guide students to the solution of problems with which they had a lot of trouble. Ask if there are any further questions, answer them and continue to the next activity.

Find the formula for the lateral surface area of the rectangular prism as a class, using the following as a guide for discussion:

> To find the lateral surface area of this rectangular prism, students need to add the areas of the front and back and the left and right sides. The front and back have the same area, as do the left and right. So the equation for surface area should look something like $SA = 2(8 \cdot 4) + 2(6 \cdot 4)$. When the students examine this formula with only variables, it should look like $SA = 2(l \cdot h) + 2(w \cdot h)$. Factor the height out of the right-hand-side to get $SA = h(2l + 2w)$ which is simply the height multiplied by the perimeter of the base. The formula for lateral surface area then, is $SA = Ph$ where $P$ is the perimeter of the base and $h$ is the height of the rectangular prism.




The students will then return to their partners from Day One. They will examine their work from Day One and analyze the steps needed to find the lateral surface area of each of the figures. They will write out the steps without substituting any numbers into their equation. The students should then try to find similar formulas for the remaining three-dimensional figures they studied on Day One. They should find that they are able to get the lateral surface area of the other prisms and the cylinder in the same way. And the pyramids? Instead of the height of the pyramid, the perimeter of the base is multiplied by the slant height.

After finding the formulas for the lateral surface area, have the students create formulas for the total surface area. These should be easy to create, since all you have to do is add on 2 multiplied by the area of the base for the prisms and cylinder and just the area of the base for the pyramid onto the formulas for the lateral surface area.

**Homework: Students will complete a set of problems finding both the lateral and total surface areas of various cylinders, prisms, and pyramids with only drawings of the 3-D figure as a reference (no nets should be drawn for the students for this assignment).**

**Day Three:**
Objectives: Students will be able to
- compute the surface areas of various three-dimensional figures.
- describe various real-world applications of surface area.

Warm-up: Which method is easier for you to find surface area: using the net of a figure and adding together the areas of the different faces or using the formula for finding the surface area or the given figure? For the method you find easier, explain the steps you take to compute the surface area.

While the students are answering the warm-up question, collect the homework from the students. Review the problems from the homework and answer any questions the students have about it.

Students will then solve various real-world situations dealing with surface area. After they finish this work, they will create their own surface area problem using a real world application along with the solution to the problem with all the work done to solve the problem. This problem will potentially be on their quiz over surface area on Day Four and/or on their Cumulative Test at the end of the unit. With any time left in class, have students show their problems to the class.

**Day Four:**
Answer any last-minute questions the students have about surface area.
**Give the students a quiz over surface area allowing the students to solve the problems using the method with which they feel the most comfortable.**

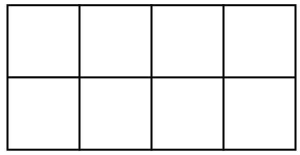
**Days Five & Six:**
Objectives: The student will be able to:
- recognize the relationship between surface area and volume.
- use different viewpoints of three-dimensional figures made up of unit blocks in order to determine the figures' volumes
- calculate the volumes of prisms, cylinders, cones, and pyramids.

Warm-up: Think again about the doghouse you want to build for you would like to build for your new puppy. What other factors are important to consider when building your doghouse? What changes might you make to the doghouse if you decided to buy a Toy Poodle, as opposed to a German Shepherd? Why?

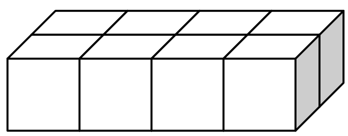
While the students are answering the warm-up questions, check the homework, grading on effort (if the students have shown their work and tried to solve the problems or have written questions or explanations when they got stuck). After the students have had a few minutes to answer the warm-up questions, have the students write the solutions to the homework problems on the board and explain the solutions to the class. Clear up any misconceptions along the way.

Then, discuss the responses to the warm-up questions (introducing the terms surface area and volume, if the terms have not already been used), ask the questions: Does a smaller surface area always mean a smaller volume? Can you have the same surface area for two different figures, yet have the same volume? Allow students to answer without any affirmations or negations to their answers; only record their answers. To answer explore these questions further the students need to have a better understanding of volume. Begin with something simple, the notes on the following page as a guide:

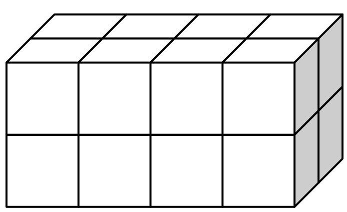
If each of the squares comprising this rectangle has side lengths of one unit, what is the area of the rectangle?

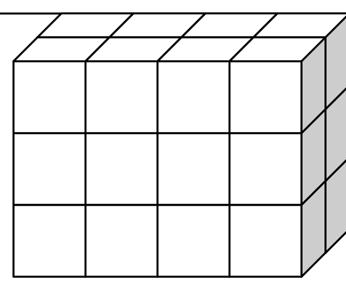


Now consider that this rectangle is the top view of a rectangular prism. When might someone look at only the top view (or side view or a corner view, etc.) of an object? (Bring up topics like blueprints, maps; sculptors may like to look at their sculptures from different viewpoints). It is often important in those cases to know how much space an object takes at hand. How do we do that? What is the volume of the rectangular prism is the height of the prism represented above is one unit?



What is the volume of the rectangular prism if the height of the prism is two units? three units? What's a quick way of computing the volume (without adding all the cubes together)? Write a formula for this quick method. Will this method work for cylinders and other prisms? Is there a way to divide a circle or triangle into one by one squares? Does it make sense if there aren't all full squares? How do you solve that problem?





The next step is to demonstrate that the area of a pyramid is one-third of the area of its corresponding prism. Using plastic manipulatives, have students fill various pyramids with water; then use that water to fill up their corresponding prisms. What conclusions can be drawn? Is there a similar figure for cylinders? What are the similarities and differences between pyramids and cones? Point out that the surface area and volume for cones can be found using the same formulas as they would for pyramids.

**Homework: Students will compute the volumes of the figures from the surface area worksheet.**

**Day Seven:**
Warm-up: Give the students 2-3 real-world problems using volume.

While students are working on the warm-up problems, check the homework for effort. The students will display the warm-up problem solutions and the homework solutions on the board. The students will spend the rest of the time in pairs solving real-world problems using volume. As they did with surface area, students will create their own problems (accompanied by the solutions) that could be used for the volume quiz and/or on the cumulative test. Any extra time will be spent on the presentation of these problems.

**Day Eight:**
Answer any last-minute questions the students have about volume.
**Give the students a quiz over volume.**

**Day Nine:**
Objectives: Students will be able to:
- discover the effects that changes in surface area have on volume and volume changes on surface area.
- explore the effects that an object's shape has on its surface area and volume.

Warm-up: What is the relationship between surface area and volume?

Allow students to consider the answer to this question; then refer back to the answers the students gave on Day Five for the following questions: Does a smaller surface area always mean a smaller volume? Can you have the same surface area for two different figures, yet have the same volume?

Discuss their thoughts now compared to those they had before? Tell the students to scrutinize the surface area and volume homework assignments they completed. Some of the students may have noticed that they used the same figures, others may have not. The students should now not only different prisms to each other, but also prisms to pyramids, cylinders to prisms, etc. What conclusions can be formed? What patterns exist?

Students will be given a packet with all the eligible problems they created themselves (both for surface area and volume) some of which may be from other class periods. This will be given as a review for their cumulative test. They are to start solving the problems after the discussion has ended. This packet will be for their own use and does not have to be turned in.

**Homework: Students will write their reflections on the day's discussion. Were they surprised by any outcomes, for example? Students will also write explanations on the steps needed to solve surface area and volume problems, how they recognize surface area and volume problems, and why they are important.**

**Days Ten – Eleven:**
Objectives: Students will be able to:
- apply the knowledge they have regarding surface area and volume to design a new container for their favorite M&M's.
- construct a new container for the M&M's
- compare and contrast different three-dimensional figures to find the "best" container.
- put themselves "in the shoes" of a manufacturer, store owner, and consumer in order to figure out which of their proposed containers would be best to produce.

These days will be set aside for the students to work on their projects. The teacher will help the students as they need it and make sure they are working in the right direction. Students will finish anything they didn't finish in class for homework.

**Days Twelve & Thirteen**
Objective: Students will be able to:
- assume the role of a designer addressing a marketing advisor to sell his/her product

**Students will present their product to the class and turn in all necessary paperwork and their finished container.**

**Day Fourteen:**
Students will complete a self-reflection on their project answering such questions as:
1) What do you like most about your project?
2) What would you have liked to change about your project?
3) What challenges did you face while completing this project?

The rest of the class will be spent as a review for the cumulative test.

**Day Fifteen:**

**Cumulative Test**

Name: ______________________________
Date: ____________________
Period: ________

**Day One: Introduction to Nets and Surface Area**

Draw three different nets for each figure. Then, find the lateral and total surface areas for each of the figures.

**Rectangular Prism:**

| Net 1: | Net 2: | Net 3: |
|---|---|---|
| | | |

Lateral Surface Area:

Total Surface Area:

**Triangular Prism:**

| Net 1: | Net 2: | Net 3: |
|---|---|---|
| | | |

Lateral Surface Area:

Total Surface Area:

**Rectangular Pyramid:**

| Net 1: | Net 2: | Net 3: |
| --- | --- | --- |

Lateral Surface Area:

Total Surface Area:

**Triangular Pyramid:**

| Net 1: | Net 2: | Net 3: |
| --- | --- | --- |

Lateral Surface Area:

Total Surface Area:

**Cylinder:**

| Net 1: | Net 2: | Net 3: |
|---|---|---|
| | | |

Lateral Surface Area:

Total Surface Area:

**Pentagonal Prism:**

| Net 1: | Net 2: | Net 3: |
|---|---|---|
| | | |

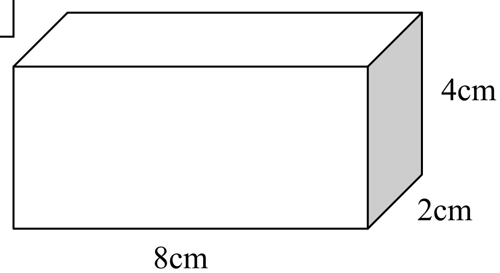
Lateral Surface Area:

Total Surface Area:

Name: ___________________________
Date: __________________
Period: _____

Surface Area Homework

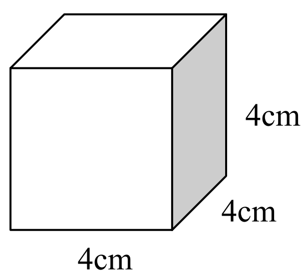
1.




Lateral Surface Area: __________

Total Surface Area: __________

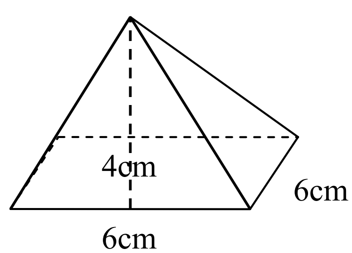
2.




Lateral Surface Area: __________

Total Surface Area: __________

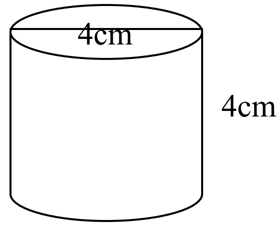
3.




Lateral Surface Area: __________

Total Surface Area: __________

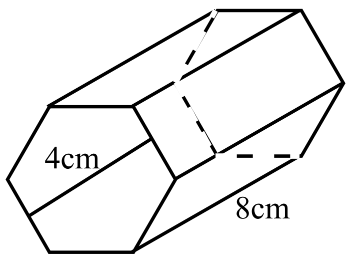
4.




Lateral Surface Area: __________

Total Surface Area: __________

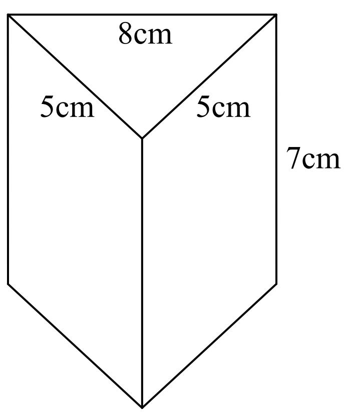
5.




Lateral Surface Area: __________

Total Surface Area: __________

6.




Lateral Surface Area: __________

Total Surface Area: __________

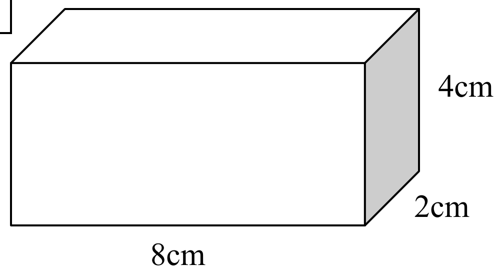
Name: ____________________________

Date: ___________________

Period: ______

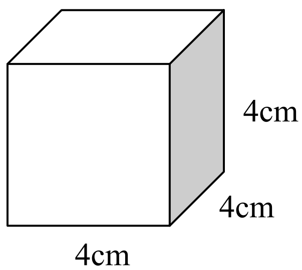
Volume Homework

1.



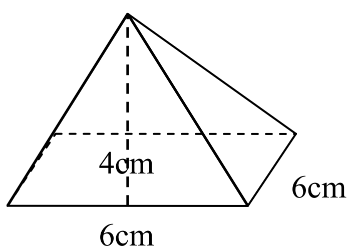

Volume: __________

2.



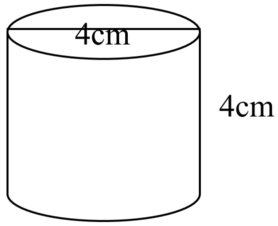

Volume: __________

3.



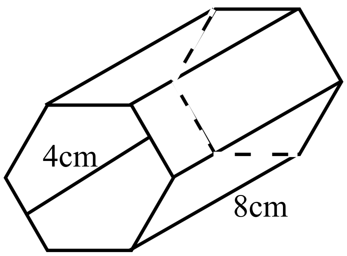

Volume: __________

4.



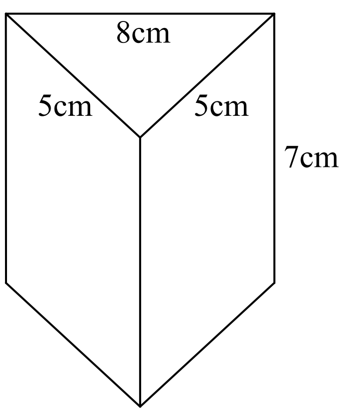

Volume: __________

5.




Volume: __________

6.




Volume: __________

Name: ___________________________

Date: _____________________

Period: _________

Surface Area and Volume Project

Due: __________

The marketing advisor for M&M's has decided that a new container is needed for M&M's and you have been hired to create it. You are to choose between cylinders, prisms, and pyramids for the new look. You need to take into consideration all the different items that must go on the labels for your container including, but not limited to, the logo, company name, & nutritional facts. Also, in order to make the best decision for your shape, you must compare different designs and figure out each design's strengths and weaknesses. Which container is best for the manufacturer? the store owner? the consumer? Why? Which container do you think would be the most visually appealing? Based on all your considerations, which container will you present?

Now that you've gotten that settled, you need to present your final product. Create a prototype: create the container, design the label for it, and fill it with M&M's. Create a visual (poster, PowerPoint presentation, etc.) showing why this is the best choice for the company. Make your presentation.

Checklist:

- Finished container.
- Visual for the presentation.
- Page(s) with all mathematical calculations and explanations of those calculations.
- Page(s) displaying comparisons between different possible containers and an explanation regarding the container selected. Why did you choose that container?
- Page with the design specifications of your container so the manufacturer could look at it and know exactly how to make it.

| | Fails to Meet Expectations | Approaches Expectations | Meets Expectations | Exceeds Expectations |
|---|---|---|---|---|
| Comparison | The student gives no comparison, has limited points of distinction, or the points of distinction are not clearly stated or explained. | The student gives a comparison of two figures with 3-4 points of distinction stated with a very short and inadequate explanation. | The student gives a comparison of three figures with five points of distinction clearly stated and explained. | The student gives a comparison of four or more figures with more than five points of distinction clearly stated and explained thoroughly. |
| Finished Container | Very little or none of the information that needs to be on the container is there. The container looks unpolished and has no visual appeal. | Most of the information that needs to be on the container is there. The container shows very little creativity and has little appeal. | All the information that needs to be on the container is there. The container is visually appealing but the information is not well-organized (crowded, hard to read, etc.) | All the information that needs to be on the is there. The container is visually appealing and the information is well-organized. |
| Mathematical Calculations | None of the mathematical calculations that were used in the comparisons are given or they are not correct. There is no explanation given for any comparison. | Some of the mathematical calculations that were used in the comparisons are given mostly correct. Steps are not shown in a logical progression. They are also not written neatly or labeled clearly and very poor explanation is given. | All of the mathematical calculations that were used in the comparisons are given. They information is correct and logical, but each step is only is succinctly explained. The process can be followed, but could leave questions about your process. | All of the mathematical calculations that were used in the comparisons are given, logical, and correct. They are written neatly and labeled clearly and each step is thoroughly explained. |
| Design Specifications | No net drawing of the container is given with side lengths/dimensions labeled. The desired place of opening is not clearly shown. No diagrams of the container are given. | A net drawing of the container is given with some side lengths/dimensions labeled. The desired place of opening is not clearly shown. Fewer than three views of the container are shown. Drawings are not only not drawn to scale, they also look quite unpolished. | A net drawing of the container is given with all necessary side lengths/dimensions clearly labeled. The desired place of opening is clearly shown. Three views of the container are shown. Drawings are not drawn to scale. | A net drawing of the container is given with all necessary side lengths/dimensions clearly labeled. The desired place of opening is clearly shown. Four or more views of the container are shown and are drawn to scale and are very precise. |
| Visual for Presentation | The visual is not easy to comprehend and does not reinforce your decision to choose the container you chose; there is no explanation for your reasoning . | The visual only slightly reinforces your decision to choose the container you chose; some of your reasoning is explained, but other key explanations are not given. | The visual reinforces your decision to choose the container you did as opposed to the other containers you researched; your explanations are present, but they are very short. | The visual is easy-to-read, visually appealing, and strongly reinforces your decision to choose the container you did as opposed to the other containers you researched. Your reasoning is thoroughly explained. |