6-2016

# Maps and Distance in Geometry (9th-10th grade)

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| **Unit Title:** Maps and Distance in Geometry | **Grade Level:** 9th and 10th Geometry |
| **Subject/ Topic Area:** Geometry | **Topics:** Points, Lines, and Planes; Midpoints; Distance; Maps |
| **Time Frame:** Two weeks | **Designed By:** Catherine Davison Sporer |
| **School:** The International School of the Americas | **School District:** North East ISD, San Antonio TX |

**Abstract:**

The purpose of this unit is to make mathematical concepts more concrete and real for students using maps. Students will be asked to think about mathematical objects through cartography or map-making. When teaching the formulas for midpoint and distance, I have found that more concrete understanding of the topic helps students to understand and apply the process for midpoints and distances. A distance between two arbitrary coordinate points is more difficult for students to grasp conceptually than the distance between San Antonio and Dallas or the distance from school to their favorite restaurants.

Students will begin the unit with a pre-assessment, in which they attempt to find distances and midpoints on a map without formulas or a grid and explain their process. On the pre-assessment day, students will also learn about map distortion and think about how a spherical surface turns into a flat map by projecting through a sphere with a flashlight. Teachers will also use a map to introduce the terms point, line, and plane. Students will continue learning these basic terms of geometry with a kinesthetic activity and discover the midpoint and distance formulas through investigation activities. After practicing with the formulas, students will return to maps of ancient civilizations and use their learning of the formulas to calculate distances and find midpoints on maps. After calculating simple straight distances, students will look at more complicated paths or multi-leg journeys, similarly to paths created on apps like Google Maps. Students will mathematically develop their own "Google Maps" with multi-leg and straight path journeys.

This unit contains two quick quizzes or checks for understanding over the math skills. The unit culminates in a performance assessment in which students use the features of a map to determine the length of a trip that they design given constraints of distance and cost.

**Teacher Notes:**

In this unit, the chosen maps focus primarily AP World History Curriculum and could be replaced depending on the grade level of the students. Maps can be printed and copied onto graph paper or an online tool can be used to draw a grid onto an image.

## Stage 1 – Desired Results

(2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:
(A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;
(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and

(4) Logical argument and constructions. The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to:
(A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems.
(5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:
(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships;

### Transfer

*Students will independently use their learning to…*

Determine the effectiveness of solutions.
Solve problems arising in the world with accuracy.

### Meaning

**Understandings**

*Students will understand that….*

Modeling with mathematics can help me to understand complex topics.
The world can be modeled with points, lines, and planes.
There are multiple ways to approach and solve the same problem.
Math can be used to communicate ideas and to provide evidence for statements.

**Essential Questions**

How can spatial information be communicated effectively?
How do I know that a solution is best and accurate?
How do I know what information to include and what I should leave out?
Why is the utilization of formulas important?

### Acquisition

**Knowledge**

*Students will know…*

- The definitions of the undefined and defined terms: point, line, plane, line segment, ray, opposite, rays.
- The distinction between defined and undefined terms.
- The meanings of the geometry terms: endpoint, midpoint, distance, segment bisector, collinear, non-collinear, coplanar, and non-coplanar.
- The language of cartography including the terms: latitude, longitude, coordinates, and key.
- Examples of the vocabulary words: point, line, and plane.
- The formulas for midpoint and distance.

**Skills**

*Students will be able to…*

- Find the distance and midpoint between points on a number line.
- Find the coordinates of a point in the coordinate plane.
- Use the distance formula to find the distance between two points with and without a coordinate plane.
- Use the midpoint formula to find the midpoint between two coordinate points with and without a grid.
- Use the distance formula to find the length of a path.
- Overlay a coordinate plane onto a map and use the scale of the map to determine the distance between location points.
- Solve problems when operating under real-world constraints.

Learning Goals:
1.1: I can define the basic defined and undefined terms in geometry and provide examples from diagrams.
1.2: I can find distances and midpoints of segments on number lines. I can construct the midpoint of a segment.
1.3: I can find distances in the coordinate plane using the distance formula.
1.4: I can find midpoints in the coordinate plane using the midpoint formula.
1.5: I can use distance and midpoint to solve real world problems.

## Stage 2 – Evidence

| **CODE** (M or T) | **Evaluative Criteria** (for rubric) | |
|---|---|---|
| a) M<br>b) T<br>c) M<br>d) T<br>e) T | a) Explains thinking clearly<br>b) Mathematically describes and models a real world situation<br>c) Understands and operates under constraints<br>d) Supports arguments and justifies ideas using mathematics<br>e) Reflects on effectiveness of solutions | Performance Task(s)<br>*Students will demonstrate meaning-making and transfer by…*<br>Task 1: The Dream Trip:<br>1) Creating a trip with at least five stopping points across Europe, researching and justifying the significance of the stopping points.<br>2) Estimating and calculating the distance of the trip.<br>3) Making adjustments to the trip based on constraints of time, money, and distance. Explaining the need for these adjustments and why they were chosen.<br>4) Comparing their results to real data. Reflecting on and explaining the successes and shortcomings of their mathematical procedure. Identifying any constraints that were missing from the problem.<br>---<br>Other Evidence (e.g., formative)<br>Short practices<br>Quick Quizzes |

## Stage 3 – Learning Plan

| **CODE** (A, M, T) | Pre-Assessment<br>*How will you check students' prior knowledge, skill levels, and potential misconceptions?* |
|---|---|
| | For the pre-assessment, provide students with the map of Myanmar (if students are in 9th grade, connections could be made to their learning in World Geography classes). Provide students with a straight edge and tell them that they are permitted to draw on the map. This map should have no overlay. Have students answer the following questions:<br>1) What is the meaning of the word distance?<br>2) Try to find the distance between Naypyidaw and Bangkok. Describe your process for finding the distance.<br>3) Try to find the distance between Naypyidaw and Bangkok using a second difference process. Describe your process for finding the distance. |

| | | |
|---|---|---|
| | 4) What is the meaning of the word midpoint?<br>5) Which city looks like it is near the midpoint of Naypyidaw and Bangkok? How did you determine that this city was the midpoint?<br>6) How could the process of finding the midpoints and distances on a map be made easier? What materials would you need? | |
| Day 1: M, T | **Learning Activities:** *(Italicized items are provided at the end of the document)*<br>Day 1: How are maps made?<br>*1) Pre-Assessment (pages 8-9)*: Students use the map of Myanmar to answer the questions about distance and midpoint.<br>2) Unfolding the Earth: Pose the question to students: How do we turn a globe into a map? Open the discussion of the difficulty of turning a spherical surface into a flat one. What is the best way to make a map? Provide student table groups with spherical paper lanterns or paper globes and a pair of scissors. Give students the challenge of making the best flat map from the paper spherical object. Which group was most successful?<br>3) Projections Activity: Introduce the vocabulary of latitude, longitude, and projection. Demonstrate the creation of a projection by projecting a flash light through a clear curved object with some design on it. One recommendation is to use part of a plastic soda bottle with some design drawn on it. Place the soda bottle or object on a blank piece of paper. Hold the flash light above the object and ask a few students to help you draw the projected image on the paper. Discussion Questions: What happens to the image when it is projected? How is size and shape preserved? How is size and shape distorted? Revisit the discussion questions after presenting students with images of various earth projections, such as the Mercator and the Robinson projections.<br>4) *Exit Ticket (page 10):* Students answer short reflection questions on the creative of maps on their exit tickets. | Progress Monitoring (e.g., formative data)<br>Day 1:<br>Exit ticket: Constraints/ Difficulties of Map-Making |
| Day 2: A | Day 2: Points, Lines, and Planes and Vocabulary<br>(Learning Goal 1.1)<br>1) *Points, lines, and Planes on a Map (page 11):* Each student receives a copy of the "US Map with Roads and Rivers". Using this map prompt students through the discussion questions through which they will begin to define points, lines, and planes. Points are "locations" in space without a size or distance. Lines are any type of "path" on the map, like rivers, boundaries, or roads. Note that when you redefine these later, lines must be straight and extend infinitely. A Plane is a surface.<br>2) Kinesthetic activity and *Notes (page 12):* Define Points, Lines and Planes terms with students. Give each student a small jar of play-doh. Ask students to try and create each object as you define it. Use this powerpoint or create your own to work through the definitions. Provide students with the | Day 2:<br>Short Practice: Points, Lines, Planes |

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| | graphic organizer provided in this document on which they provide<br>3) *Practice 1.1: Points Lines and Planes (page 13)* : In the last few minutes of class or for homework, students complete the short practice. | |
| Day 3: A | Day 3: Distance and Midpoints on Lines<br>(Learning Goals 1.1&1.2)<br>1) Check Practice 1.1<br>*2) Points, Lines, Planes on the Walls Warm Up (page 14):* Before class, tape points on the walls of the classroom in the same positions as the diagram for students. The purpose of this warm up is to give students a visual for understanding points, lines, and planes in three dimensions. Having a paper diagram that matches what they see on the classroom walls will help them to conceptualize these terms.<br>*3) Notes: Distance and Midpoints on Number Lines (page 15-17)*<br>4) Practice 1.2: Number Lines. Provide students with mixed practice over Points, Lines, and Planes (similar to Practice 1.1) and finding distance and midpoints from number lines (similar to the examples from Notes: Distance and Midpoints on Number Lines). | Day 3:<br>Short Practice: Number Lines |
| Day 4: A, M | Day 4: The Pythagorean Theorem and deriving the distance formula<br>(Learning Goal 1.3, assess Learning Goals 1.1&1.2)<br>*1) Map Warm Up (page 18):* Provide students with a copy of the Myanmar map (same as the pre-assessment) and with a straight-edge and compass. Students will use the map to find midpoints by construction and distance by creating their own number line. Have students draw on the map. Students connect the cities Naypyidaw and Bangkok on their maps. First they will construct the midpoint using the process from the notes on Day 3. After constructing the midpoint, students will measure the scale of the map and create a "number line" between the two cities, where the notches on the number line match the distance on the scale of the map. Students will use the number line that they draw to determine the distance between the two cities and miles and the midpoint between the two cities. Which city is closest to the constructed/calculated midpoint?<br>*2) Quick Quiz 1.1&1.2 (page 19)*<br>*3) Pythagorean Investigation and Distance Formula Notes (page 20-21)*: Review the Pythagorean Theorem and provide students with several examples of the theorem, finding the hypotenuse of a right triangle. Complete the same problems on top of a coordinate grid and use the distances on the grid and the learning from number lines and distance to develop the distance formula. After | Day 4:<br>Quick Quiz: Points, Lines, Planes<br>Short Practice: Distance Formula |

| | | |
|---|---|---|
| | developing the formula, complete several problems of finding the distance between two points with and without a graph.<br>4) Practice 1.3: Distance Formula: Assign problems for practice with a mixture of problems that have a graph (like examples 2 and 3) and problems that do not have a graph (like examples 4 and 5). | |
| Day 5: A, M | Day 5: Deriving the midpoint formula<br>(Learning Goal 1.4)<br>1) Check Practice 1.3<br>*2) 1.4 Midpoints Notes: (page 22-23):* Revisit finding midpoints on number lines with the warm up example in the notes. Discuss the meaning of midpoint as the "average" of two endpoints. What if the line is diagonal? How do you find the "average" if you have both x and y coordinates? Present example 1 on a grid, where the midpoint turns out to be a whole number. Derive the formula from example 1, using the idea that you need the middle or average of the x's as well as the middle or average of the y's, so your midpoint will have two parts. Test that the formula works for example 1. For examples 2 and 3 complete the scaffolded examples in the notes with graphs and check that the midpoint examples make sense on the graphs. For examples 4 and 5, find midpoints without using graphs and only with the formula.<br>3) Practice 1.4: Midpoints. Assign a practice that includes both finding midpoints from graphs and from coordinate points not on graphs. | Day 5:<br>Short Practice: Midpoint Formula |
| Day 6: A,M | Day 6: Distance and Midpoint Practice<br>(Learning Goals 1.3&1.4)<br>*1) Coaching Practice (page 24-25)* : This exercise was developed from a strategy called "Reciprocal Learning" in *The Strategic Teacher.*[i] In this exercise, students work in pairs. Each pair of students has one whiteboard and marker and one set of cards. The cards are in pairs with the same problem labeled "Coach" and "Player." The "Player" card contains only the problem to solve. The "Coach" card contains the problem, as well as steps to solve the problem, hints for the problem, and the answer. Students take turns with their partner in the roles or Player and Coach. The Player works out the problem on the whiteboard, while the Coach assists the Player with the problem and checks answers when the play is finished with the problem.<br>*2) Map Problems (page 26-27):* Using a map of Europe with a grid overlay, students decide on a coordinate system of the map based on the grid (which point is 0,0?). Using the map, they will start by finding distance between various cities, and then find the midpoints. | Day 6:<br>Practice Cards (self-check) |

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|---|---|---|
| | 3) Practice for 1.3&1.4: The teacher can assign additional practice problems for tomorrows quiz if students need more practice over midpoint and distance. | |
| Day 7: M, T | Day 7: Finding the Length of Paths on Maps<br>(Learning Goal 1.5, assess Learning Goals 1.3&1.4)<br>*1) Quick Quiz 1.3&1.4 (page 28)*<br>2) Google Maps Warm Up: Ask students to use their phones or computers to input a local location- restaurant, another school, or their houses. After they input this destination, ask them what they notice about the paths that Google calculated to their destinations. Are the paths straight? How many pieces does the path have?<br>3) A Path over Land Migration Problem (page 29): Provide students with the world map and migration paths. If students are in 10th grade, tie in their learning about early human migration from their World History Class. On the map, as a class choose a coordinate plane and a starting point in Central Africa. Choose an ending point in Asia and try to create a straight leg path from the starting to the ending point. Students can use a compass to try to find the stopping points. Students should draw the straight-leg path when they have it, then begin to calculate the distances of each leg of the path.<br>4) Practice 1.5: Another Migration Problem. For practice, assign students another path on the World Migration map. For students that have shown advanced proficiency assign a longer path (to South America), and for students that have shown a developing proficiency assign a short path (to Europe). | Day 7:<br>Quick Quiz: Distance and Midpoint<br>Short Practice: Migration Problem |
| Days 8-9: M, T | Days 8-9: *Performance Assessment (page 30-32):* Through this Performance Assessment, students will create a trip in which they calculate distances and midpoints and think about the constraints that go into a trip: obstacles, money, distance, time. | Days 8-9:<br>Performance Assessment,<br>Written Reflection |

Name:_____ Period:_____ Date:_____

Using the map of Myanmar and a straight edge, answer the following questions as completely as possible:

1) What is the meaning of the word distance?

2) Try to find the distance between Naypyidaw and Bangkok. Describe your process for finding the distance.

3) Try to find the distance between Naypyidaw and Bangkok using a second difference process. Describe your process for finding the distance.

4) What is the meaning of the word midpoint?

5) Which city looks like it is near the midpoint of Naypyidaw and Bangkok? How did you determine that this city was the midpoint?

6) How could the process of finding the midpoints and distances on a map be made easier? What materials would you need?


Image: https://en.wikipedia.org/wiki/Outline_of_Myanmar

Day 1: Exit Ticket

Name:_____ Period:____

1. Describe the challenges and shortcomings of maps and of the creation of maps that we discussed today?

2. What might be some other challenges related to maps and the creation of maps that we did not discuss today?

Day 1: Exit Ticket

Name:_____ Period:____

1. Describe the challenges and shortcomings of maps and of the creation of maps that we discussed today?

2. What might be some other challenges related to maps and the creation of maps that we did not discuss today?

Day 1: Exit Ticket

Name:_____ Period:____

1. Describe the challenges and shortcomings of maps and of the creation of maps that we discussed today?

2. What might be some other challenges related to maps and the creation of maps that we did not discuss today?


1. Which objects on this maps would be considered planes?

2. How could we define points based on what they mean on a map?

3. Which objects on this map would be considered lines?

4. How could we define lines based on what they mean on a map?

5. Which objects on this map would be considered planes?

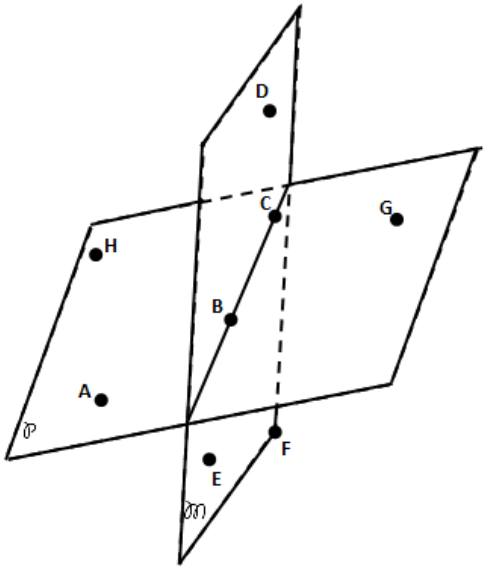
6. How could we define lines based on what they mean on a map?

Image: https://commons.wikimedia.org/wiki/Atlas_of_the_United_States

## 1.4 Points, Lines, and Planes (and More!) Notes

| Object | What is it? | What does it look like? | How do we name it? | Examples |
|---|---|---|---|---|
| Point | | | | |
| Line | | | | |
| Plane | | | | |
| Segment | | | | |
| Ray | | | | |

| Term | What does it mean? | What does it look like? |
|---|---|---|
| Collinear | | |
| Coplanar | | |




Name all points that lie on Plane $\mathcal{P}$:

_____. These points are_____.

Name all points that lie on Plane $\mathcal{M}$:

_____. These points are_____.

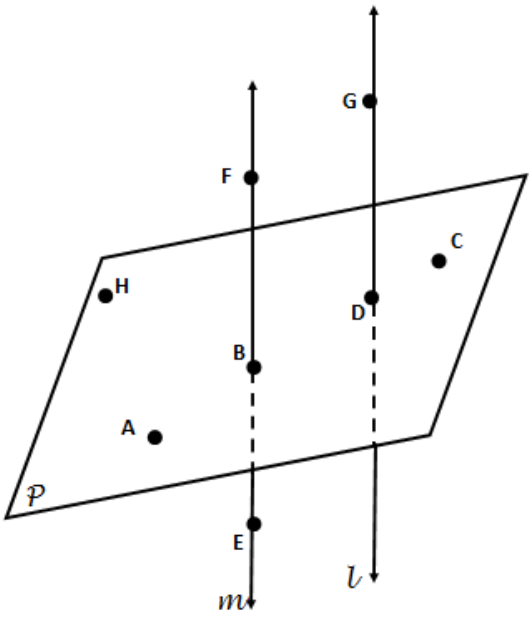
Name two points that are collinear:_____

Name Plane $\mathcal{P}$ two different ways:_____

Day 2: 1.1 Practice

Name:_____ Period:_____

In the diagram below Plane $\mathcal{P}$ is intersected by line $m$ and line $l$. Answer the following questions

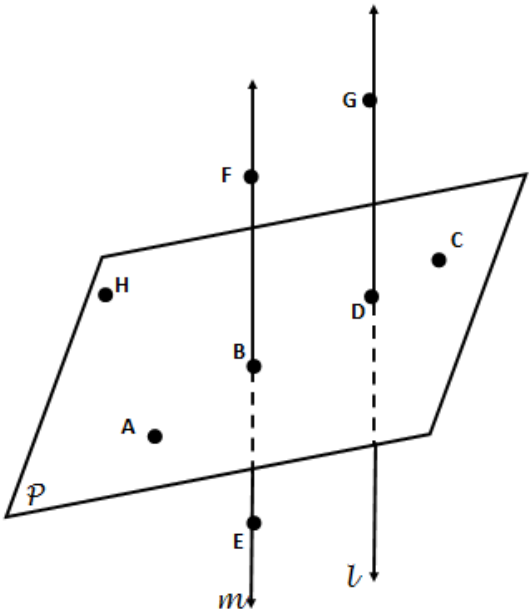



1. Name a point the is collinear with point D:_____
2. Name a point that is collinear with point F:_____
3. Name all points that lie on plane $\mathcal{P}$ :_____
   These points are _____.
4. Name plane $\mathcal{P}$ in two different ways: _____
5. Name line $m$ in two different ways:_____
6. When you think about maps, what might the world collinear mean on a map?

Day 2: 1.1 Practice

Name:_____ Period:_____

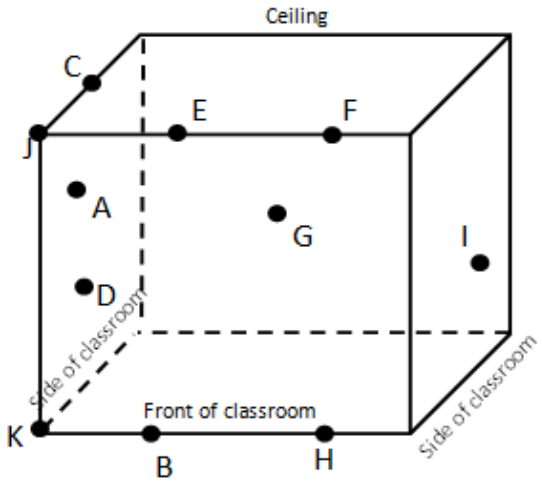
In the diagram below Plane $\mathcal{P}$ is intersected by line $m$ and line $l$. Answer the following questions




1. Name a point the is collinear with point D:_____
2. Name a point that is collinear with point F:_____
3. Name all points that lie on plane $\mathcal{P}$ :_____
   These points are _____.
4. Name plane $\mathcal{P}$ in two different ways: _____
5. Name line $m$ in two different ways:_____
6. When you think about maps, what might the world collinear mean on a map?

Day 3: Points, Lines, and Planes on the Walls

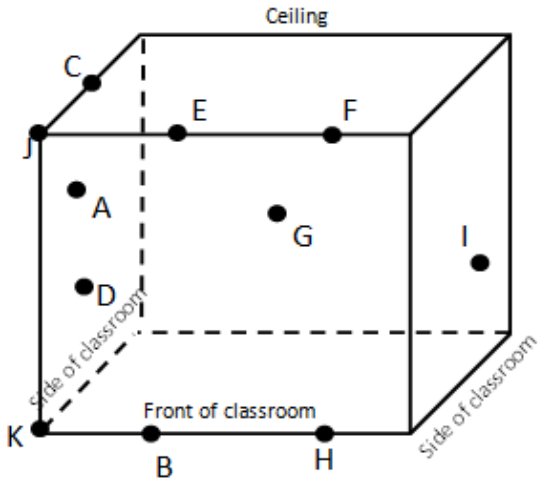
Name:_____ Period:_____




1. What is the intersection of the lines JK and BH? _____
2. What is the intersection of the front of the classroom and the side of the classroom (the side that contains point D)? _____
3. Name a point the is collinear with point B and H:_____
4. The side of the classroom could be named plane CAD. What is another way of naming this plane?_____
5. What is the intersection of the lines EF and JC? _____
6. Name the plane at the front of the classroom. _____
   What is another way of naming this plane? _____
7. What is another way to name the line JE _____
8. What is the intersection of the ceiling of the classroom and the side of the classroom (the side that contains point D)? _____

Day 3: Points, Lines, and Planes on the Walls

Name:_____ Period:_____




1. What is the intersection of the lines JK and BH? _____
2. What is the intersection of the front of the classroom and the side of the classroom (the side that contains point D)? _____
3. Name a point the is collinear with point B and H:_____
4. The side of the classroom could be named plane CAD. What is another way of naming this plane?_____
5. What is the intersection of the lines EF and JC? _____
6. Name the plane at the front of the classroom. _____ What is another way of naming this plane? _____
7. What is another way to name the line JE _____
8. What is the intersection of the ceiling of the classroom and the side of the classroom (the side that contains point D)? _____

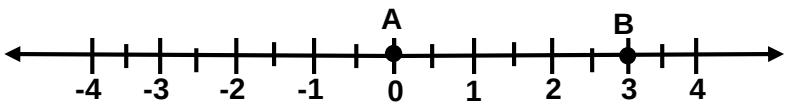
Day 3 Notes: Distance and Midpoints on Segments

## 1.2 Distance and Midpoints on Number Lines Notes

### VOCABULARY:

| | What is it? | What does it look like? |
|---|---|---|
| Line Segment | _____ and all of the _____ in between. | |
| Equidistant | | |
| Bisector | | |
| Midpoint | A MIDPOINT on a segment that is _____ from both _____. It lies on the _____ of a segment. | |

For example, AB is a line segment on the number line below.



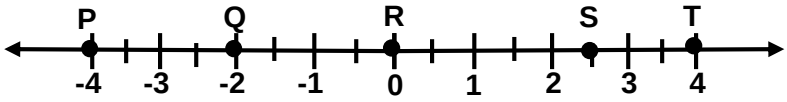
What is the length of the segment AB?

The length of a segment is the distance between two endpoints, and is represented by the letters of the two endpoints. The distance from A to B is represented AB. Distance is ALWAYS POSITVE!

The length of a segment can be found by _____ or by _____, so

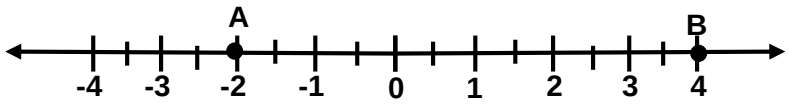
On a number line,

$$\text{Distance} = x_2 - x_1$$




Using the number line below, find the following:

| | | |
|---|---|---|
| PQ= | QT= | PT= |
| RT= | QR= | PS= |



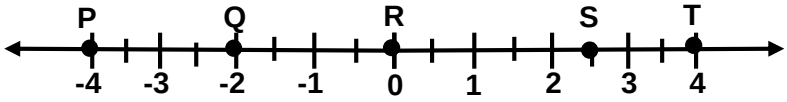

What is the midpoint of the segment AB?

How do you know?

The midpoint is like the "AVERAGE" of the two endpoints. How do you find an average?

On a number line,

$$\text{Midpoint} = \frac{x_1 + x_2}{2}$$



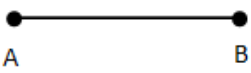
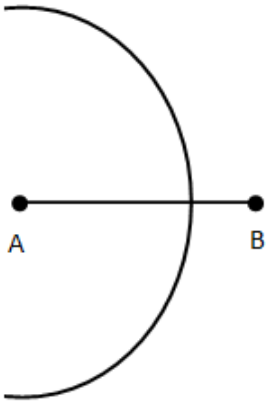
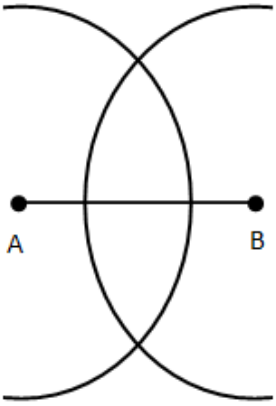
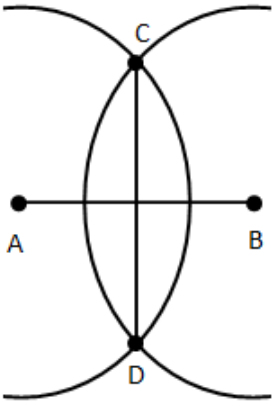

Using the number line below, find the midpoints of the following (decimals are OK!):

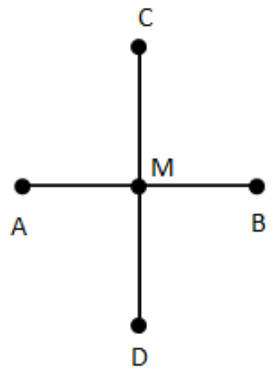
| | | |
|---|---|---|
| P and Q: | Q and T: | P and T: |
| R and T: | Q and R: | P and S: |

If we do not have coordinates or numbers, we can find a midpoint by construction, using a straight edge and a compass. Follow the steps below:

## Constructing Midpoints:

Given two points, A and B, construct the midpoint:


| | |
|---|---|
| Step 1: Draw a segment connecting A and B. |   |
| Step 2: Placing your compass point at A, extend the compass so that it is more than half-way from A to B, and draw an arc. |   |
| Step 3: Keep the compass at the same length. Draw an arc with that compass length that is centered at B. This arc should intersect your first arc one time above and one time below your segment AB. |   |
| Step 4: Draw the points C and D at the intersections of the arcs, and connect C and D with a line segment |   |

| | |
|---|---|
| Step 5: Draw the point at the intersection of AB and CD. This point is the midpoint! Call it M. |   |

Construct the midpoint between A and B below:


Name:_____ Period:_____ Day 4: Maps Warm Up


1. Use the midpoint construction that we learned yesterday to construct the midpoint between Bangkok and Naypyidaw. (Hint: You need a straight-edge and a compass, and you should start by connecting the two points with a line segment!)

2. Which city is closed to your constructed midpoint?_____

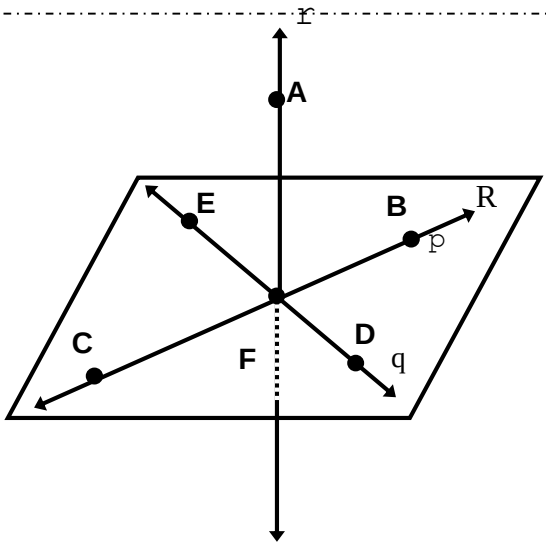
3. Use the scale in the bottom right and a straight edge to create a "number line" like we had in class yesterday. Use your number line to find the distance between Bangkok and Naypyidaw in miles._____

4. Use your number line to find the midpoint between Bangkok and Naypyidaw. How well does this match with your constructed midpoint? What is the distance between the midpoint and Bangkok? What is the distance between the midpoint and Naypyidaw?

5. Find the distance two other cities by creating a number line and using the scale of the map: Chosen cities:_____ Distance:_____

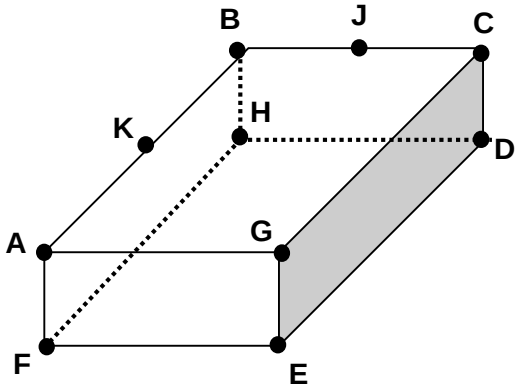
| **Good luck to:** | **Skills: 1.1 &1.2:** 1.1: I can define the basic defined and undefined terms in geometry and provide examples from diagrams. 1.2: I can find distances and midpoints of segments on number lines. I can construct the midpoint of a segment. |
|---|---|

FIGURE 1



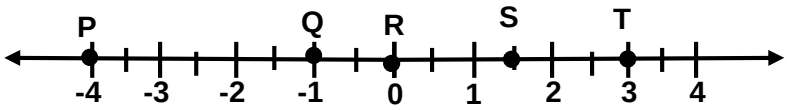

1. Name the point that is collinear with A.
2. Name two points that are coplanar with B and E.
3. What is another way to name line $\overleftrightarrow{CB}$ ?

FIGURE 2




4. Name the two planes that intersect at line $\overleftrightarrow{GE}$

Use the number line to answer 5-8.

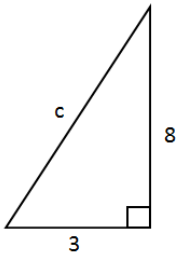



| | |
|---|---|
| 5. Find the distance QT= | 6. Find the midpoint between P and T: |
| 7. Find the midpoint between Q and T: | 8. Find the distance QS= |

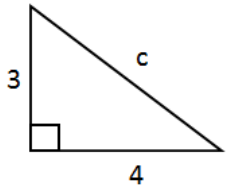
## 1.3 The Pythagorean Theorem & the Distance Formula Notes

Recall the Pythagorean Theorem: $a^2 + b^2 = c^2$, where a and b are the lengths of legs of a right triangle and c is the length of the hypotenuse of the right triangle. Find the hypotenuse c in each of the triangles below:

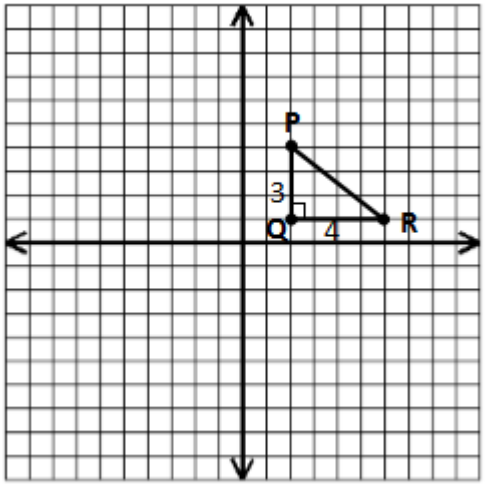
Ex 1:




Ex 2:




What if the triangle in example 2 is placed over a grid? Do the lengths of the legs change?




What are the coordinates (x,y) of the points?

P:_____ call it $(x_P, y_P)$

Q:_____ call it $(x_Q, y_Q)$

R:_____ call it $(x_R, y_R)$

What is the length of QR?_____

Which values of the Q and R points are the same? x or y?

What is another way we could find the length QR?

What is the length of AB? _____

Which values of the P and Q points are the same? x or y?

What is another way we could find the length AB?

Using the Pythagorean Theorem, we could find PR, the hypotenuse, $a^2 + b^2 = c^2 \rightarrow c = \sqrt{a^2 + b^2}$.

$c = \sqrt{a^2 + b^2}$ because _____

$PR = \sqrt{QR^2 + PQ^2}$ because _____

$PR = \sqrt{(x_R - x_Q)^2 + (y_Q - y_P)^2}$ because _____. Is there any way to get rid of $x_Q$ or $y_Q$? What are these numbers the same as?

$PR = \sqrt{(x_R - x_p)^2 + (y_R - y_P)^2}$ because _____.

Now plug in the coordinate numbers of the points to make sure that this works! Does this formula work???

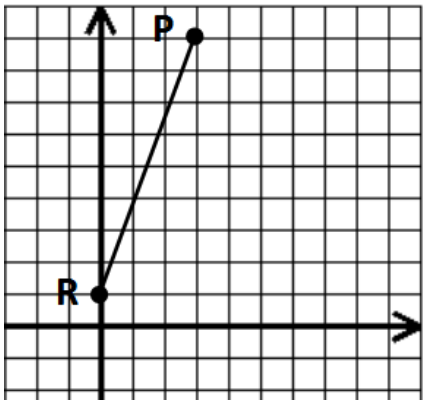
Earlier we found that $PR = \sqrt{(x_R - x_p)^2 + (y_R - y_P)^2}$. The general formula for any distance is in a coordinate plane is:

$$\textbf{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where in this formula, $(x_1\ ,\ y_1)$ is the first point (that's why it has little number 1's!)

and $(x_2\ ,\ y_2)$ is the second point (that's why it has little number 2's!)

Try ex 1 again, this time on the coordinate plane. Find the length of PR by plugging into the distance formula, then use your answer from example 1 to check. Why is this the same as ex 1?




$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

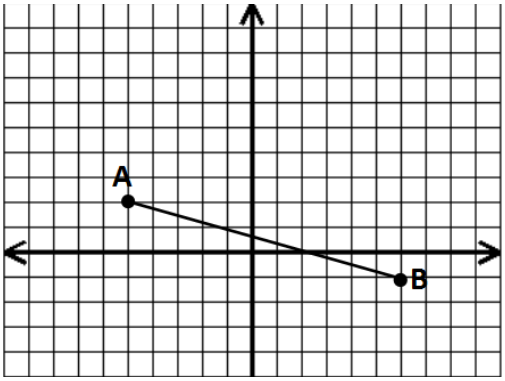
P is point 1 $(x_1\ ,\ y_1)$. What are the coordinates of P?

R is point 2 $(x_2\ ,\ y_2)$. What are the coordinates of R?

Plug into the formula:

$$PR = \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2}$$

Ex 3: Find the length of AB




$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

A is point 1 $(x_1\ ,\ y_1)$. What are the coordinates of A?

B is point 2 $(x_2\ ,\ y_2)$. What are the coordinates of B?

Plug into the formula:

$$AB = \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2}$$

Ex 4: Find the length on TU if the coordinates of T are (4,0) and the coordinates of U are (6, -5)

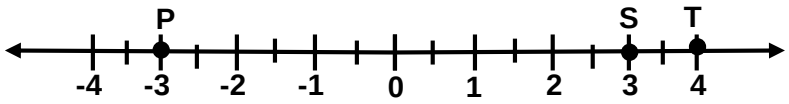
$$TU = \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2}$$

Ex 5: Find the length on CD if the coordinates of C are (-5,-2) and the coordinates of D are (8, 10)

$$CD = \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2}$$

1.4 Midpoints Notes

Remember: The midpoint is like the "AVERAGE" of the two endpoints.



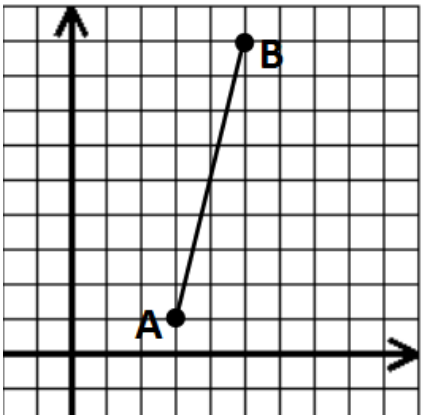

What is the midpoint between S and T?

What is the midpoint between P and T? Remember you can use the number line midpoint formula:

Midpoint = $\frac{x_1 + x_2}{2}$ , which calculate the average between P and T.

**What if the line is diagonal? How do you find the "average" if you have x and y coordinates?**

Ex 1: Find the midpoint of the segment AB (the point that would cut AB exactly in half!)




What point looks like it is in the middle?

Draw a right triangle with AB as the hypotenuse.

Find the "middle" of the bottom leg of the triangle:

Find the "middle" of the other leg of the triangle:

What is the "middle" of the hypotenuse?

A midpoint is a point! So it has two parts: an x-coordinate and a y-coordinate.

What are the coordinates of A? _____ This is the first point: $(x_1, y_1)$

What are the coordinates of B? _____ This is the second point: $(x_2, y_2)$

To find the x-coordinate of the midpoint, find the middle (average) of the x's → $\frac{x_1 + x_2}{2}$ look familiar?

To find the y-coordinate of the midpoint, find the middle (average) of the y's → $\frac{y_1 + y_2}{2}$ SO…

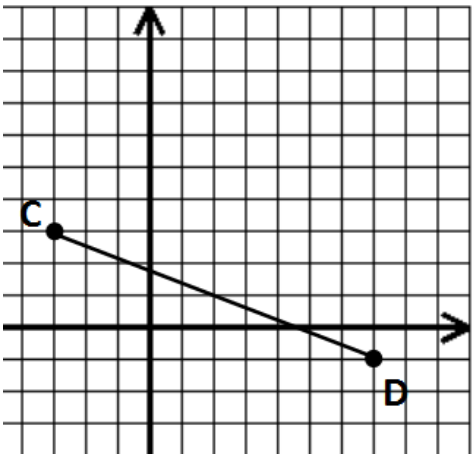
Midpoint = (middle of X's , middle of Y's) ← Two Parts!!!!

OR **Midpoint =** $\left(\frac{x_1 + x_2}{2} \quad , \frac{y_1 + y_2}{2}\right)$ ← TWO PARTS!

Find the midpoint of AB in Ex 1 using this formula: $\left(\frac{\_\_ + \_\_}{2} \quad , \frac{\_\_ + \_\_}{2}\right)$

**Midpoint =** $\left(\frac{x_1 + x_2}{2} \quad , \frac{y_1 + y_2}{2}\right)$

Ex 2: Find the midpoint of CD:

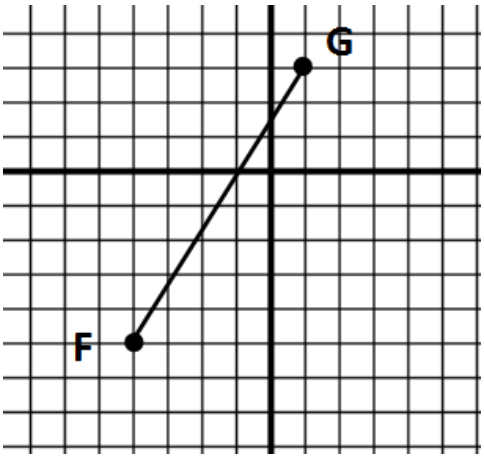



Find the coordinates of C: _____ and of D:_____

$\boldsymbol{Midpoint} = \left(\frac{\_ + \_}{2} \quad , \frac{\_ + \_}{2}\right)$

Check it on your graph, does this midpoint make sense?

Ex 3: Find the midpoint of FG




Find the coordinates of F: _____ and of G:_____

$\boldsymbol{Midpoint} = \left(\frac{\_ + \_}{2} \quad , \frac{\_ + \_}{2}\right)$

Check it on your graph, does this midpoint make sense?

Ex 4: Find the Midpoint of TU if the coordinates of T are (4,0) and the coordinates of U are (6, -5)

$Midpoint = \left(\frac{\_ + \_}{2} \quad , \frac{\_ + \_}{2}\right)$

Ex 5: Find the length on CD if the coordinates of C are (-5,-2) and the coordinates of D are (8, 10)

$Midpoint = \left(\frac{\_ + \_}{2} \quad , \frac{\_ + \_}{2}\right)$

| PLAYER #1 | COACH #1 |
|---|---|
| Find the distance between (2,3) and (-1,7) | Find the distance between (2,3) and (-1,7)<br>Label the points $(x_1, y_1)$ $(x_2, y_2)$<br>Use the distance formula:<br>Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$<br>Distance $= \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_ - \_\_\_)^2}$<br>Plug the labeled points into the blanks.<br>Distance $= \sqrt{(-1 - 2)^2 + (7 - 3)^2}$<br>$= \sqrt{(-3)^2 + (4)^2}$<br>$= \sqrt{9 + 16}$<br>$= \sqrt{25}$<br>$= 5$ |

| PLAYER #2 | COACH #2 |
|---|---|
| Find the distance between (1,5) and (-3,10) | Find the distance between (1,5) and (-3,10)<br>Label the points $(x_1, y_1)$ $(x_2, y_2)$<br>Use the distance formula:<br>Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$<br>Distance $= \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_ - \_\_\_)^2}$<br>Plug the labeled points into the blanks.<br>Distance $= \sqrt{(-3 - 1)^2 + (10 - 5)^2}$<br>$= \sqrt{(-4)^2 + (5)^2}$<br>$= \sqrt{16 + 25}$<br>$= \sqrt{41}$<br>$= 6.40$ |

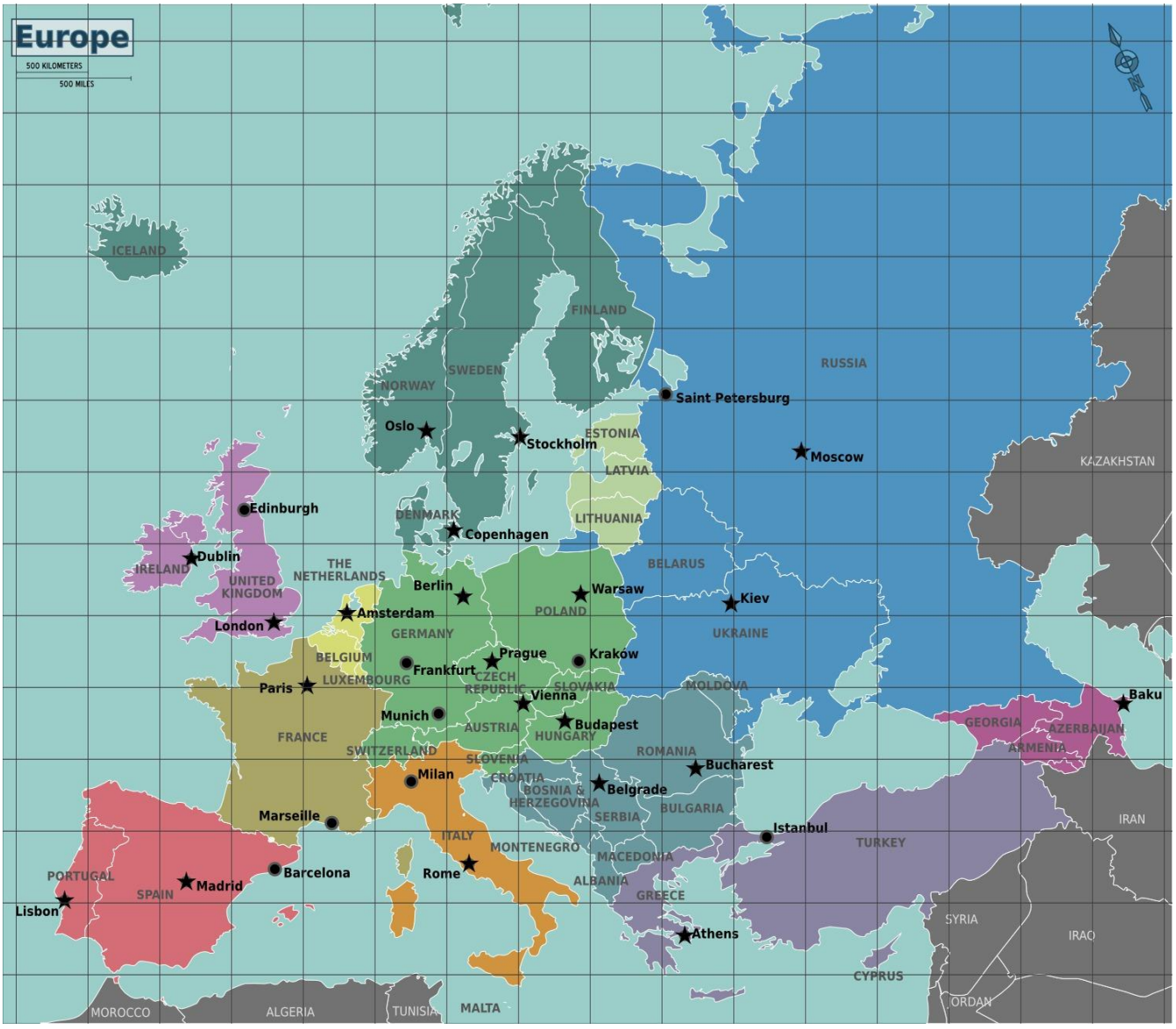
| PLAYER #3 | COACH #3 |
|---|---|
| Find the midpoint between (4,5) and (-2,8) | Find the midpoint between (4,5) and (-2,8)<br>Label the points $(x_1, y_1)$ $(x_2, y_2)$<br>Use the midpoint formula:<br>**Midpoint** = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$<br>**Midpoint** = $\left(\frac{\_\_ + \_\_}{2}, \frac{\_\_ + \_\_}{2}\right)$<br>Fill in the blanks.<br>**Midpoint** = $\left(\frac{4 + -2}{2}, \frac{5 + 8}{2}\right)$<br>= $\left(\frac{2}{2}, \frac{13}{2}\right)$<br>= $(1, 6.5)$ |

| PLAYER #4 | COACH #3 |
|---|---|
| Find the midpoint between (0,-7) and (3,-1) | Find the midpoint between (0,-7) and (3,-1)<br>Label the points $(x_1, y_1)$ $(x_2, y_2)$<br>Use the midpoint formula:<br>**Midpoint** = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$<br>**Midpoint** = $\left(\frac{\_\_ + \_\_}{2}, \frac{\_\_ + \_\_}{2}\right)$<br>Fill in the blanks.<br>**Midpoint** = $\left(\frac{0 + 3}{2}, \frac{-7 + -1}{2}\right)$<br>= $\left(\frac{3}{2}, \frac{-8}{2}\right)$<br>= $(1.5, -4)$ |

Name:_____ Period:___ Day 6: Maps Problems

| Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | **Midpoint** = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ |
|---|---|

The map of Europe below has a scale of 500km per unit on the graph. The first thing we need to determine is where to put the x-axis and the y-axis.




1. Using this coordinate system, what are the coordinates of Saint Petersburg? _____ $(x_1, y_1)$
   What are the coordinates of London?_____ $(x_2, y_2)$
   Use the distance formula to find the distance between Saint Petersburg and London:

$$\text{Distance} = \sqrt{(\_\_ - \_\_)^2 + (\_\_ - \_\_)^2}$$

This distance is in units, multiply by the scale of 500 km to find the actual distance:_____

2. Using this coordinate system, what are the coordinates of Stockholm? _____ $(x_1 , y_1)$
   What are the coordinates of Paris?_____ $(x_2 , y_2)$
   Use the distance formula to find the distance between Stockholm and Paris:

   Distance $= \sqrt{(\_\_ - \_\_)^2 + (\_\_ - \_\_)^2}$

   This distance is in units, multiply by the scale of 500 km to find the actual distance:_____

3. What city looks like it is closest to the midpoint of Lisbon and Moscow?
   Use the midpoint formula to find the actual midpoint between Lisbon and Moscow:

   Find the coordinates of Lisbon_____ $(x_1 , y_1)$

   Find the coordinates of Moscow _____ $(x_2 , y_2)$

   **Midpoint =** $\left(\frac{\_\_\_ + \_\_\_}{\mathbf{2}} \quad , \frac{\_\_\_ + \_\_\_}{\mathbf{2}}\right)$

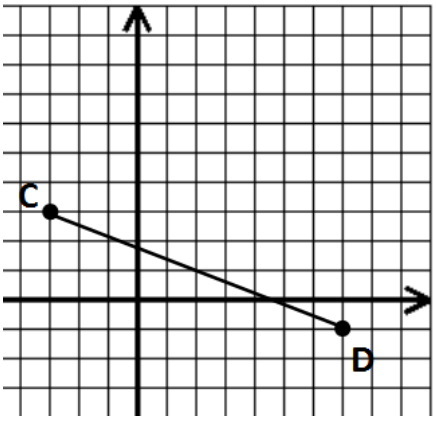
   Which city is closest to the actual midpoint?

Image: https://upload.wikimedia.org/wikipedia/commons/a/a4/Europe_regions_minimal_cities.png

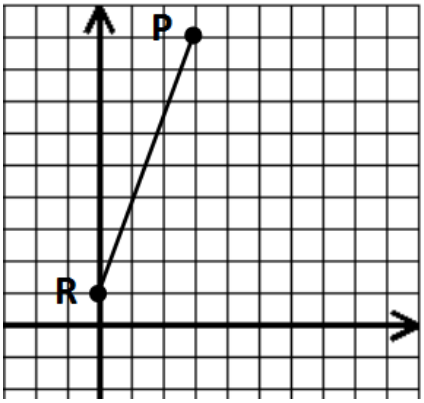
Name:_____ Period:_____

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Midpoint =** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

1. Find the length of the distance CD in the graph below.




2. Find the midpoint Between R and P.

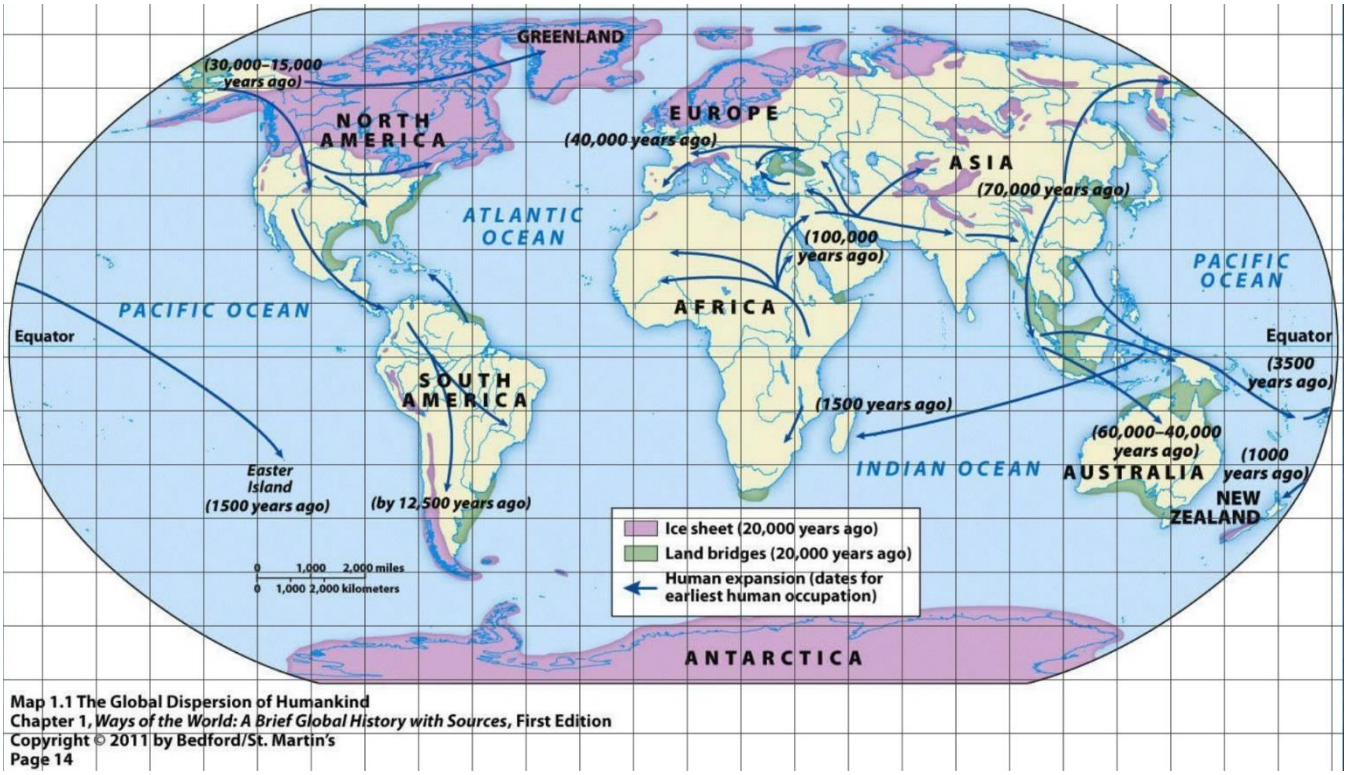



3. Find the distance between (0,-5) and (5, 7)

4. Find the midpoint between (-10, 3) and (2, 7)

Name:_____ Period:_____ Day 7: Migration Problem

On the world graph below, the scale is 1,000 miles per unit. First, we need to draw on the x-axis and the y-axis. We will be creating a path from the starting point of East Central Africa.



Map 1.1 The Global Dispersion of Humankind
Chapter 1, *Ways of the World: A Brief Global History with Sources*, First Edition

Page 14

1. Draw a point at the start of the arrows in Africa. What are the coordinates of this point? _____
   Draw the ending point in Australia at the end of the arrow. What are the coordinates? _____
   What is the straight distance between Africa and Australia?

2. Determine a path from Africa to Australia with two stopping points (stopping point A and stopping point B) along the way. We can only travel in straight lines! Draw the stopping points on the map.
   What are the coordinates of Stopping Point A _____
   What are the coordinates of Stopping Point B _____

   Leg 1 of the journey: Find the distance between the starting point in Africa and Stopping Point A:

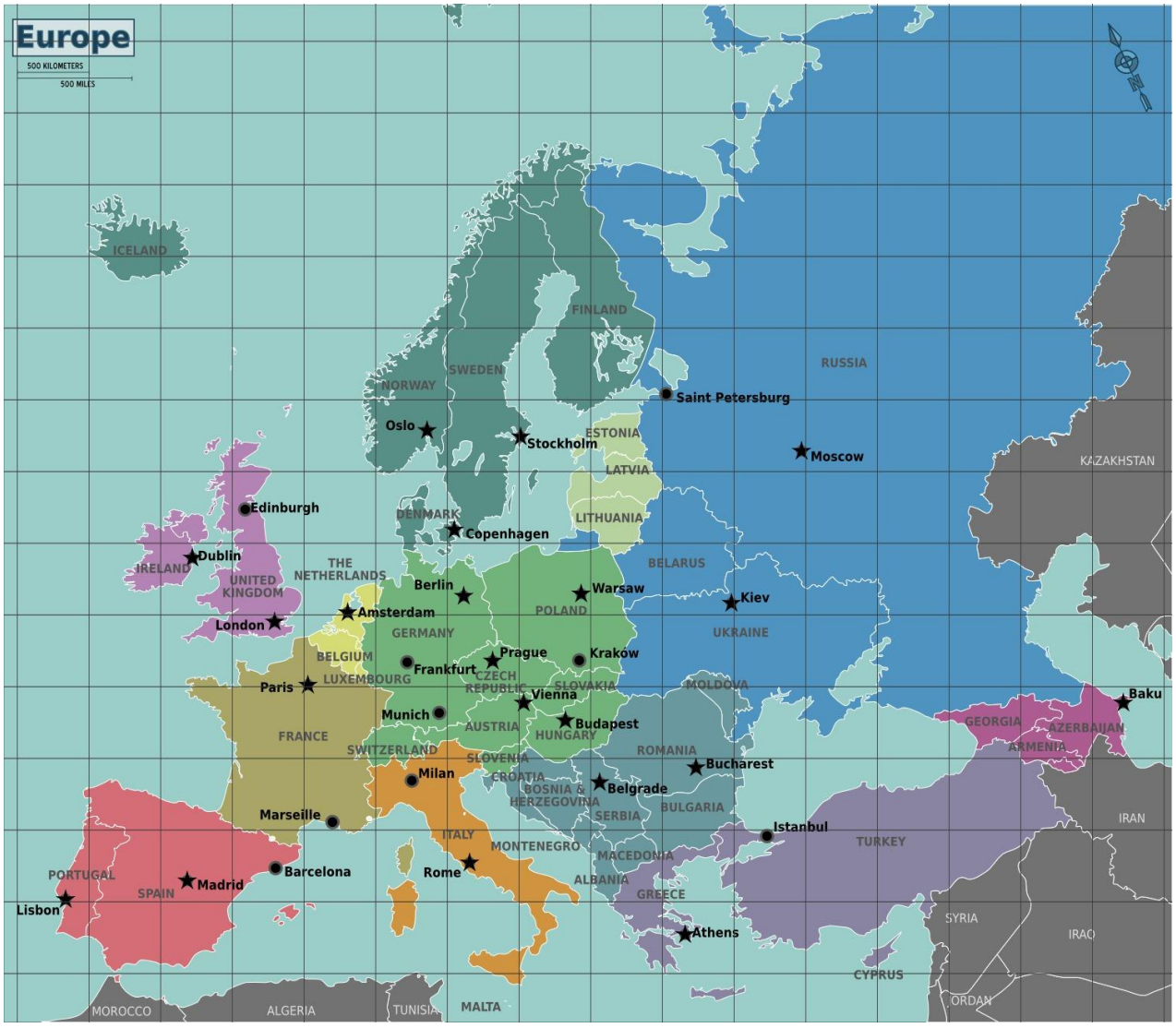
   Leg 2 of the journey: Find the distance between Stopping Point A and Stopping Point B:

   Leg 3 of the journey: Find the distance between Stopping Point B and the end Point in Australia:

   Add the lengths of these three legs together to get the total length of the journey in units.

You will use the map of Europe with scale of 500km per unit to plan a trip. Your trip plan must include:

- At least 5 "Stopping Points" including the starting and ending point of your choice. These points should be cities that you find significant due to their history or personal interest to you.
- A distance plan that is between 3000 km and 7000 km in length.
- An analysis of the cost and time that it would take to complete your trip.




1) Choose the cities that will be the starting and ending point of your trip, and find their coordinates. Describe the historical or individual significance of each. Why do you want to visit this place? Why is this an important location? (May require some research and citations).
Starting Point:_____ Coordinates:_____

Describe the significance of this location:

Ending Point:_____ Coordinates:_____

Describe the significance of this location:

2) Draw the straight distance between your starting and ending point on the map. Calculate the straight distance between the starting and ending point using the distance formula.

3) Find the coordinates of the midpoint of the straight-leg path.

4) Choose three additional points for your trip and find their coordinates. Describe the historical or individual significance of each. Why do you want to visit this place? Why is this an important location? (May require some research and citations).

Point A:_____ Coordinates:_____

Describe the significance of this location:

Point B:_____ Coordinates:_____

Describe the significance of this location:

Point C:_____ Coordinates:_____

Describe the significance of this location:

5) Find the distances of each leg of the journey:

Leg 1: Starting Point to Point A

Leg 2: Point A to Point B

Leg 3: Point B to Point C

Leg 4: Point C to Ending Point

6) Add the leg distances together to get the total distance: _____

Multiply by the scale (500km) to get the actual distance:_____

)

Does this distance fit within the distance constraints? If not, on a separate sheet of paper, make adjustments and describe the changes.

7) Calculate an estimated cost of this trip. First, you need to think about the type of trip you would want to take (backpack, take trains, rent a car, stay in nice hotels, stay in hostels, etc. ). Describe your trip below:

8) Estimate the amount of time it would take you to complete your trip (depends on your method of travel!):

9) Estimate the cost your trip, including estimated travel, estimated food. Cite the sources of cost figures that you find at the end of this document.

10) Think about the way that we calculate distance. What are the constraints and limitations of calculating distance this way in our calculations?

11) When you think about journeys in this situation and in previous situations (Human Migration, Myanmar, etc. ), what are the constraints and limitations of travel to think about when creating a path?

Image: https://upload.wikimedia.org/wikipedia/commons/a/a4/Europe_regions_minimal_cities.png

[i] Silver, Harvey, Strong and Perini (2007). *The Strategic Teacher.* Alexandria, VA: Association for Supervision and Curriculum Development. 2007.