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Effects of Dual-Frequency Environment Exploration on Stiffness Discrimination Thresholds

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***Abstract*— Previously, excitation frequency has been found to alter perceptual discrimination thresholds of stiffness, mass, and damping. Here, we explore how the blending of two frequencies could affect the just noticeable difference for stiffness. In a perceptual experiment based on the method of adjustments, we tested participants' ability to match a reference stiffness moving at combinations of two frequencies to explore the effects on stiffness discrimination. As more of the lower frequency was added, participants' ability to accurately match the reference was hampered. Results suggest that as two frequencies are excited, the resulting perceptual thresholds are blended between the levels for the individual frequencies.**

I. Introduction

The design and control of kinesthetic haptic devices benefits from an understanding of human haptic perception of the rendered environments. Unlike simple perception of force magnitude or displacement, perception of an impedance depends on the integration of both types of information, which degrades perception as compared to force or displacement alone [1]. For simple environments where the driving point impedance corresponds to a single element (i.e., a mechanical primitive like stiffness, mass, or damping) perception has been described with classical psychophysical techniques [1]–[3]. Such techniques typically rely on describing the just-noticeable difference (JND) between two levels of the primitive using Weber's Law, which predicts that the JND of a property is proportional to the magnitude of the reference from which it can be distinguished [4]. In the physical world and especially in virtual environments (VEs) rendered through haptic devices, there is rarely a single mechanical element presented to the user. Instead, the hardware and controller dynamics create a complex impedance presented to the user, with properties that vary across the frequency spectrum [5].

The presence of multiple mechanical primitives in the closed-loop response of a haptic device (even when the desired VE contains a single primitive) is important because the presence of other properties in an environment has been shown to affect perception of the property of interest. For instance, stiffness and mass that are present in high levels can mask damping perception, increasing the damping JND [6]. There is evidence that the importance of dynamic effects depends on the motion and excited frequencies present in the operator's interaction with the VE [7]–[11]. Recently, Fu et al. proposed a modified JND law that can explain masking effects for single-frequency interactions with spring-mass-damper systems [12]: the operator's JND was found to be dependent on the entire frequency response function (FRF) of the system, not only the single spring, mass, or damper property in question.

In this work, we sought to understand how the frequency dependence of JND in a spring-mass-damper system is integrated by the human perceptual system across simultaneous interactions at multiple frequencies. We designed a perceptual experiment based on the method of adjustments to assess stiffness JND when participants moved at combinations of two frequencies. We theorized two probable outcomes: either participants would exploit the information available at the frequency with the more accurate (smaller) JND regardless of the presence of the second frequency, or the presence of the frequency with the larger JND would hamper performance, resulting in a JND at some level between the two individual frequency JNDs. In the following, we first summarize the modified Weber's law and its application to stiffness perception for open-loop impedance control in Section II. We then describe the methods of our human-participant experiment featuring multi-frequency motions in Section III. Results and discussion are presented in Sections IV-V, and we discuss the implications and future work in Section VI.

II. Background

A. Modified Weber's Law

The FRF of a system is a ratio of the displacement x and force F_u felt by the user at the driving point, given by $H(s) = F_u/x$. The positive real portion of a spring-mass-damper response corresponds to an effective stiffness, while the negative real part corresponds to an effective mass[2]. As found by Fu et al. for spring-mass-damper systems, the magnitude of the ratio of the JND of the real part of the response to the total FRF response will remain approximately constant:

$$\left|\frac{\Delta \Re H(j\omega)}{H(j\omega)}\right| = c; \tag{1}$$

they found this constant to be approximately 12.2% [12].

B. Stiffness Perception with Multi-Frequency Interactions

In this study, we employed open-loop impedance control [13], such that the user feels the VE dynamics superimposed on the device dynamics. Modeling the device properties as an effective mass M and damper B (including any reflected

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[2]Note that the driving point impedance $\mathcal{Z}(s) = F_u/\dot{x}$ used to compute effective impedances [5] is simply a position derivative away from the FRF.

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inertia or damping from the motor), the FRF for a VE consisting of a virtual spring with stiffness K_{VE} is therefore

$$H(s) = Ms^2 + Bs + K_{VE}. \tag{2}$$

The modified Weber's Law (1) predicts that the JND at a specific interaction frequency for the real part containing the virtual spring will depend on the entire frequency response, including the mass and damping of the device. If the mass is unchanged, the real-part JND in the numerator is simply the stiffness JND $\Delta K(j\omega)$, since the difference in the real portions is

$$\begin{aligned}\Delta \Re H(j\omega) &= (K_{VE} - M\omega^2) - (K_{VE} \pm \Delta K(j\omega) - M\omega^2)\\ &= \pm \Delta K(j\omega).\end{aligned} \tag{3}$$

The stiffness JND at any given interaction frequency is therefore predicted to be the constant c multiplied by the magnitude of the FRF. As frequency increases from zero (at which the constant spring force would be the only cue experienced by the operator without any velocity), the modified JND law therefore predicts that stiffness JND will be equal to

$$|\Delta K(j\omega)| = c|H(j\omega)| = c\sqrt{(K_{VE} - M\omega^2)^2 + (B\omega)^2}. \tag{4}$$

Depending on the frequencies involved and the magnitude of the mass and damping, an operator could therefore be predicted to have quite different stiffness JNDs at two different frequencies. For example, Fig. 1 shows stiffness JNDs at frequencies of 0.5 Hz and 2 Hz for two different reference stiffnesses.

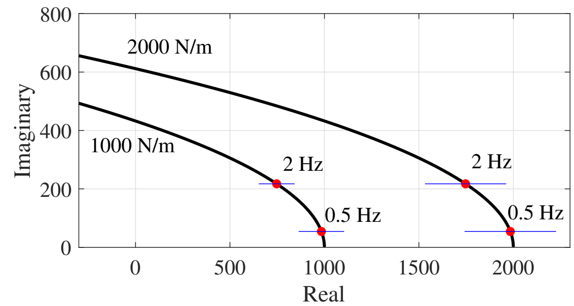

Fig. 1. Real and imaginary components of the FRF for the upper and lower limits of reference stiffness employed in this experiment. The two movement frequencies, 0.5 Hz and 2 Hz, are indicated with red dots. Blue bars indicate the JND at each frequency predicted by (1).

III. Methods

We designed a perceptual experiment in order to test how blending two movement frequencies would influence the stiffness JND. We employed a protocol based on the method of adjustments using a single degree of freedom haptic device. For each trial, a reference stiffness was randomly generated between the limits of 1000 N/m and 2000 N/m, and participants were asked to move at various combinations of two frequencies while matching the comparison stiffness to the reference. The two stiffnesses shown in Fig. 1 are these maximum and minimum reference stiffnesses, and the reference VE responses are indicated by red dots at the two motion frequencies, 2 Hz and 0.5 Hz. The blue lines in the figure indicate plus or minus the expected JND at each frequency, as predicted by (4). The JND lines for 2 Hz in both cases are shorter than the JND lines for the 0.5 Hz, predicting that there should be a difference in the JND found while looking at the participants movement at the desired frequency combinations.

Eight participants (four male, four female, all right-handed) took part in an experiment following a protocol approved by the Trinity University Institutional Review Board. Four participants had significant experience with the apparatus, while the other four participants were naïve to the device.

A. Apparatus

The experimental setup is shown in Fig. 2. The 1-D haptic device consisted of a Maxon RE65 motor attached to a Del-Tron HPS3-4 linear slide via a 1-inch capstan. Position was measured with an encoder (US Digital E2 series, 1024 counts/revolution) on the motor's back shaft. A Transducer Techniques LSP-10 load cell measured the force felt by the participant. The device has an effective mass $M \approx 1.60$ kg and a damping $B \approx 17.3$ Ns/m, based on system identification described in [14]. Control and data logging were performed using a Sensoray 626 data acquisition card with MATLAB/Simulink Desktop Real-Time. An open loop impedance controller was used to render a spring VE, and the sampling rate was set at 1 kHz.

The participant wore noise canceling headphones playing pink noise to eliminate distraction or auditory cues from the device. The participant was not able to see the device because it was blocked off by a privacy screen. The participant was able to see the screen and the stiffness adjustment controls shown in Fig. 2. The screen displayed the reference sinusoid with an overlaid plot of the user's movement in a Simulink scope window, allowing the participant to see how closely they were tracking the reference signal. Additionally, a duplicate of the position scope was created for the experimenter to monitor participant tracking performance on a

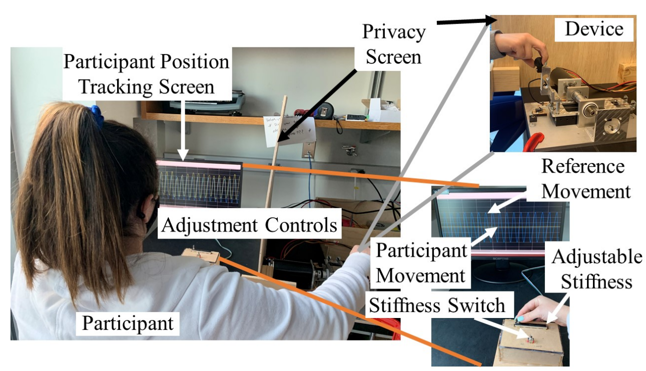

Fig. 2. Experimental set up with detail insets of the device and the participant's adjustable controls.

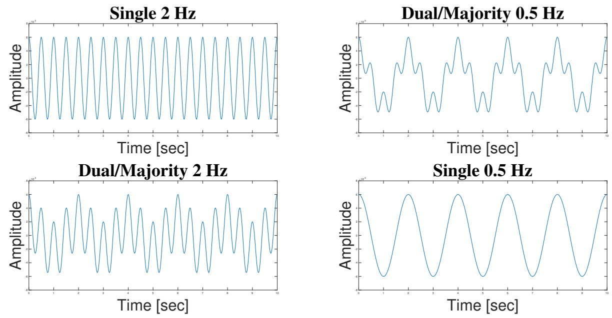

Fig. 3. Four reference signals participants were asked to track while performing the stiffness-matching task. 10 seconds of each signal is shown, corresponding with the rolling Simulink scope display participants viewed.

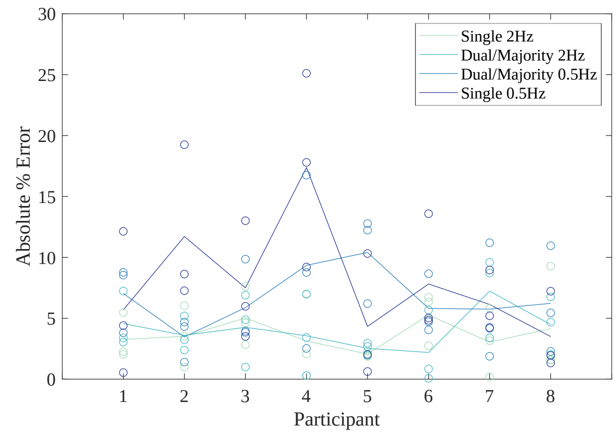

Fig. 4. Participant absolute error for each trial (circles), with lines connecting the average error of each participant. Each color represents a different condition for the reference signal.

second monitor. The stiffness adjustment controls contained a toggle switch and a linear potentiometer for the participant to complete the desired tasks.

B. Task

During each trial, the participant attempted to adjust a comparison stiffness to match the reference stiffness while exploring the two VEs according to the prescribed signal they were required to track. The reference spring stiffness ranged from 1000 N/m to 2000 N/m (randomly generated for each trial). The participant could use the toggle switch to switch between the reference stiffness and the adjustable comparison stiffness. The comparison stiffness was controlled by the participant via the linear potentiometer; for each trial, it started at 800 N/m and could be adjusted to a maximum of 2400 N/m. The participant was allowed to increase or decrease the stiffness in whatever intervals they chose, and switch between the reference and comparison stiffness as many times as they needed. Once the participant believed that the comparison stiffness matched the reference stiffness, they indicated that we could end the trial. To ensure that participants did not get unintended feedback from the transition between the reference and comparison stiffness, we required them to use the same hand they used to move the device to flip the switch and adjust the potentiometer.

While performing this task, participants were asked to always move the device in a manner to track a prescribed reference signal. In four different conditions, we asked participants to track different reference signals combining different ratios of two distinct frequencies: a single sinusoid at 2 Hz (*Single 2 Hz*), two different sums of a 2 Hz and a 0.5 Hz sinusoid, each with $2/3$ of the magnitude coming from one signal and $1/3$ from the other (*Dual/Majority 2 Hz* and *Dual/Majority 0.5 Hz*), and a single sinusoid at 0.5 Hz (*Single 0.5 Hz*). Figure 3 shows the four resulting reference signals.

At the beginning of each new condition, the participant was encouraged to practice tracking the new signal pattern, moving the device with their dominant hand; after the participant felt confident matching the movement with the reference stiffness, they proceeded to the testing phase by using the external toggle switch to change to the comparison stiffness for the first time. Each participant completed three trials for each reference signal condition, for a total of 12 trials. The order of the conditions was randomized for each participant, though the three trials per condition always remained in a block. Participants were encouraged to take breaks between trials to minimize fatigue. After each break, the participant was given time to get comfortable with moving at the desired frequency again.

C. Data Analysis

For each trial, the final comparison stiffness set by the participant was used to calculate an error between the reference and comparison stiffnesses. In experiments employing the method of adjustments, the difference threshold is typically computed as the standard deviation of the comparison stimulus level across many trials [4]; however, the time required to complete each trial in this case was prohibitive of collecting enough trials to calculate a useful standard deviation. Instead, we examined the absolute stiffness matching error in each condition as a proxy for JND, since an increase in JND would be expected to increase the mean absolute error.

To determine how well the participants tracked the reference signals to achieve the desired combinations of frequencies, a Fast Fourier Transform (FFT) of the position and force signals throughout each trial was inspected; the signals were lowpass filtered with a passband frequency of 490 Hz to eliminate aliasing. We examined the magnitude of the signal power at 0.5 Hz and 2 Hz as well as the magnitude and frequency of the two largest power magnitude peaks that were spaced at least 0.9 Hz apart. Peaks below 0.1 Hz were disregarded as low-frequency noise.

IV. Results

A. Stiffness Error by Condition

Individual percent error between the reference and comparison stiffness for each trial, as well as participants' average percent error across the three trials at each condition, can be seen in Fig. 4. The Single 0.5 Hz condition matching

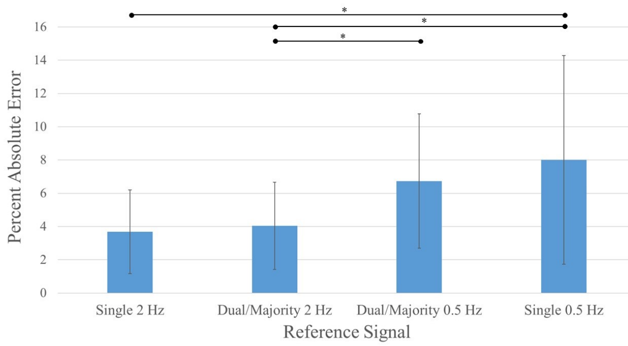

Fig. 5. Mean absolute percent stiffness error for each condition averaged across all participants and trials. Error bars represent $\pm$ 1 standard deviation. Significant differences ($p \leq 0.05$) are indicated with a * between conditions.

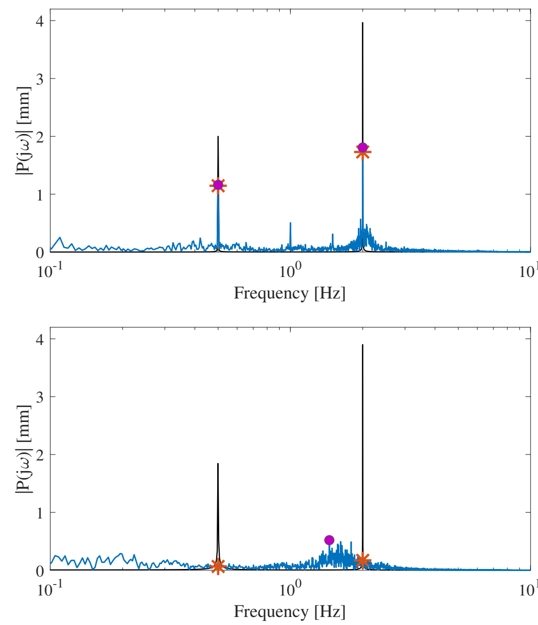

Fig. 6. Examples of experimental frequency spectra for two trials of the Dual/Majority 2 Hz condition. Spectral power is shown for the reference signal (black) and the measured position signal (blue). Red stars indicate the achieved signal power at 0.5 Hz and 2 Hz and purple circles indicate the actual measured frequency peaks. (Top): a trial in which the participant was able to accurately achieve movement at both frequencies. with magnitudes standing out from the surrounding frequencies. (Bottom): a trial in which the participant was unable to achieve clear motion at 0.5 Hz or 2 Hz; the dominant frequency was at 1.45 Hz, and the next most powerful peak was omitted as low-frequency noise (0.01 Hz).

errors were higher than the Single 2 Hz errors for all but one individual (participant 8). Additionally, there was no trend in percent absolute error observed that distinguished the naïve participants (numbered 4-7) from the participants who were familiar with device (1-3 and 8).

Figure 5 shows the average percent absolute errors across all participants in each condition. A repeated-measures (within participant) analysis of variance (ANOVA) on percent absolute error revealed significant effects by condition ($p = 0.0037, F(2.12, 48.75) = 6.12$) with a Greenhouse-Geisser correction to adjust for lack of sphericity. A post-hoc multiple comparison test revealed significant differences at a 95% confidence level between the Single 0.5 Hz and Single 2 Hz conditions ($p < 0.01$), between the two different Dual conditions ($p = 0.028$), and between the Dual/Majority 2 Hz and Single 0.5 Hz conditions ($p = 0.037$). The Single 2 Hz vs. Dual/Majority 0.5 Hz conditions approached, but did not reach, significance ($p = 0.07$).

The average percent absolute error of the first, second, and third trial across participants and conditions was calculated in order to check for learning effects in reference signal tracking. The absolute errors for trials 1-3 were all similar, at 5.73%, 5.02%, and 6.11%, respectively, indicating that difficulty tracking for each new condition likely did not play a role in the results.

B. Frequency Content

A sample of the frequency spectra for two representative Dual trials can be seen in Fig. 6; one example shows successful production of the two frequencies, while the other shows motion dominated by only a single frequency. A summary of participants' dominant movement frequencies for each condition can be found in Table I, including the mean frequency with a magnitude peak near each reference frequency as well as the standard deviation of those frequencies across all participants and trials. The largest errors in movement frequencies occurred at the 2 Hz frequency during the conditions with more 2 Hz content (Single 2 Hz and Dual/Majority 2 Hz). The high error (13.36%) for the 0.5 Hz frequency in the Dual/Majority 2 Hz condition is actually in some ways misleadingly low, since the participants who had more trouble following both sinusoids simply did not have a peak near 0.5 Hz, and therefore did not influence this number. There were 33 trials where the participants were able to achieve two distinct frequencies peaks out of the total 48 "Dual" frequency trials; 13 out of the 15 trials where participants were unable to achieve two peaks occurred in the Dual/Majority 2 Hz condition, and most participants that were unable to achieve two peaks failed to produce significant motion near 0.5 Hz in any of their trials for this condition. The Single 0.5 Hz condition had the overall smallest frequency error of 1.4% between the desired 0.5 Hz movement and the actual dominant frequency generated by the participants.

In addition to achieving the desired frequencies, if the participant accurately tracked the reference signal, the FFT should show the same ratio of frequencies in the user's position signal as in the reference signal. In order to experimentally measure the contribution of each of the dominant frequencies, we calculated the ratio $\frac{|X(j\omega_2)|}{|X(j\omega_1)|+|X(j\omega_2)|}$ by interpolating the Fast Fourier Transform results at the exact reference frequencies, 0.5 and 2 Hz; we expected to see a ratio of 0.33 between the magnitudes in the Dual/Majority 2 Hz condition. For the Dual/Majority 0.5 Hz condition, the ideal contribution from the 0.5 Hz signal would be 0.66. The experimentally calculated ratios between the displacement signal magnitudes at the two frequencies can be seen in the

TABLE I

DESIRED AND ACHIEVED FREQUENCY CONTENT FROM THE POSITION SIGNAL. FREQUENCY ERROR WAS BASED ON DOMINANT PEAKS IN THE FREQUENCY SPECTRUM; PEAKS BELOW 0.1 HZ WERE OMITTED AS LOW-FREQUENCY NOISE. FRACTION FROM 0.5 HZ IS CALCULATED BASED ON SIGNAL POWER MEASURED AT 0.5 AND 2 HZ.

Condition	Frequency 1			Frequency 2			Fraction from 0.5 Hz		
	Mean Peak Frequency [Hz]	% Error from 2 Hz	Frequency Stdev [Hz]	Mean Peak Frequency [Hz]	% Error from 0.5 Hz	Frequency Stdev [Hz]	Ideal	Mean Actual	Stdev
Single 2 Hz	2.26	13.01	0.42	N/A	–	–	0	0.106	0.112
Dual/majority 2 Hz	2.09	4.51	0.227	0.433	13.36	0.418	0.33	0.343	0.286
Dual/majority 0.5 Hz	1.95	2.41	0.363	0.522	4.39	0.064	0.67	0.711	0.196
Single 0.5 Hz	N/A	–	–	0.507	1.40	0.030	1	0.976	0.034

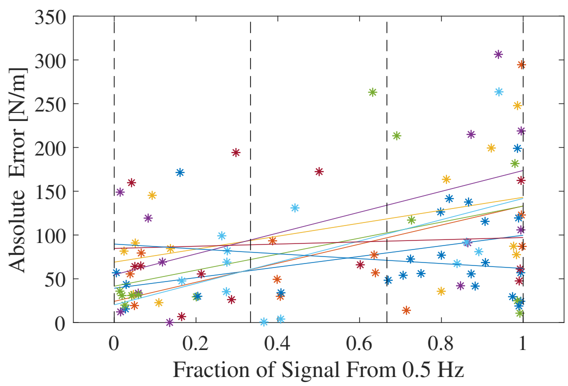

Fig. 7. Absolute error of the comparison stiffness plotted against the actual achieved ratio of movement signal power at 0.5 Hz to the combined power at 0.5 and 2 Hz. Each color represents a different participant: stars indicate the results from each trial and the solid line shows a linear fit to each participant's data. The ideal ratios for the four conditions are indicated with dashed vertical lines.

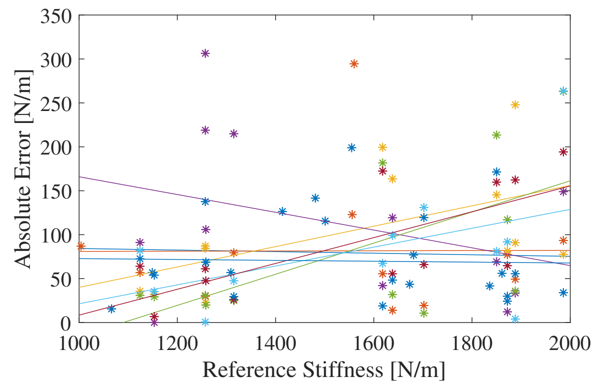

Fig. 8. Absolute error of the comparison stiffness from the reference stiffness plotted against the presented reference stiffness. Each color represents a different participant: stars indicate the results from each trial and the solid line shows a linear fit to each participant's data.

final columns of Table I labeled "Fraction from 0.5 Hz." It was seen that the average ratio of the two magnitudes was not far from the ideal for each condition (within 0.106), but the large standard deviations reflect high participant-to-participant variability, particularly in the "dual" conditions. For the single sinusoid conditions, the differences from the ideal fraction are mostly due to the discrete Fourier transform containing low power at the other frequency.

C. Stiffness Error by Actual Frequency Magnitude Ratio

The actual ratio of the frequency magnitudes for all trials were plotted against the corresponding percent absolute error of the stiffnesses, which can be seen in Fig. 7. As described above, since the discrete Fourier transform of each signal has nonzero power even at frequencies that were not excited significantly, even the single-sinusoid conditions do not achieve perfect 0 or 1 fractions. Additionally, some of the values that are shown to have a ratio near 1 or 0, which would ideally be correlated to the Single 2 Hz or Single 0.5 Hz conditions, are actually data from trials done with a Dual sinusoid reference signal when only one frequency was excited instead of two. For each participant, a linear trendline was fit to the data from their 12 trials. This revealed a positive correlation between the fraction of the signal from 0.5 Hz and the absolute percent error in stiffness for all participants except participant 8 (represented by the dark teal markers in Fig. 7), though some have shallow slopes. While not presented here, we also inspected a plot of percent absolute error vs. fraction from 0.5 Hz; little difference was observed from the results of Fig. 7, and the same seven participants still have positive linear fit slopes.

D. Stiffness Error by Reference Stiffness

The modified Weber's Law for stiffness (4) predicts that at a single excitation frequency, the JND should increase with reference stiffness. As can be seen in Fig. 8, the slope of a line fit to the error vs. reference stiffness data for each participant is either positive or quite close to zero, with the exception of participant 4 (purple).

V. DISCUSSION

The results show a relationship between the different sinusoidal combinations of motion frequencies and a participant's ability to accurately detect the stiffness of the spring. Since on average, the Single 2 Hz condition had the lowest stiffness matching error, it appears that the higher frequency exploration allowed the participants to better discern the spring stiffnesses; this agrees with what was expected based on (4), which predicted smaller JNDs at our higher (2 Hz) frequency. Additionally, the Single 0.5 Hz sinusoid had on average the largest percent absolute error out of the four combinations. Both the percentage and the raw absolute error for stiffness increased as the ratio of the 0.5 Hz to 2 Hz sinusoid increased. The Dual conditions revealed that when a 0.5 Hz motion is introduced, the percent absolute error between stiffnesses increases, though not every difference between adjacent conditions was statistically significant. Comparing Fig. 7 to Fig. 8, it appears that over the range of frequencies and reference stiffnesses explored, the effect of signal frequency content is more consistent.

Tracking performance was varied across participants; the Dual/majority 2 Hz condition in particular was difficult to follow correctly, with 5 of 8 participants failing to achieve a frequency peak near 0.5 Hz in at least one of the three trials. This likely explains why the stiffness identification errors for the Single 2 Hz condition and the Dual/Majority 2 Hz condition were so similar. Despite failure to achieve exactly the ratio in the reference signals, the Single 2 Hz, Dual/majority 0.5 Hz, and Single 0.5 Hz conditions resulted in unique movement frequency combinations, since the mean fractions from 0.5 Hz for these conditions plus or minus a standard deviation do not overlap. A limitation of our analysis is that the ratios plotted on the x-axis of Fig. 7 were based on evaluation of movement data for the entire trial, which included time when the participant was getting comfortable moving with the reference signal. We believe that in future work, we may get better results if we inspect the frequency content of the signal only towards the end of the trial when the participant is getting close to their selected final stiffness.

VI. Conclusions

Our findings build on the prior work by Fu et al. [12], which predicted that, for our experimental conditions, the stiffness JND at 2 Hz should be smaller than at 0.5 Hz. We predicted that when combining these frequencies in the exploration of a VE, either the participant would always have a JND close to the 2 Hz single-sinusoid JND when any amount of the 2 Hz signal was present in their movements or that there would be blending between the two perceptual thresholds when the second frequency is introduced, with the Single 2 Hz frequency always yielding the smallest JND. While we did not directly measure JND, our proxy (absolute error between the reference and comparison stiffness using the method of adjustments) was found to be smallest for the Single 2 Hz condition, as expected. Statistical analysis of the percent error at our four experimental conditions found significant differences in stiffness matching performance between several of the conditions. Additionally, as the 0.5 Hz signal was blended with the 2 Hz signal, we observed that the absolute error between the reference stiffness and the comparison stiffness increased—there was a positive correlation between error and the proportion of the magnitude contributed by the 0.5 Hz signal for all but one participant. This supports the hypothesis that blending between the two perceptual thresholds occurred as the two sinusoids were combined together.

While this work gives valuable insight into how humans integrate perceptual information at two different frequencies, our protocol had some limitations and additional future study is needed. While our results suggest that perceptual thresholds blend with the blended frequency content, we cannot conclude from these results whether that relationship is linear, logarithmic, or something else. Additionally, in order to better examine the coupling between reference stiffness and movement frequency effects on stiffness JND, further study is needed. Thus far, we have examined only combinations of two frequencies, and another natural question is how perceptual thresholds at three or more (up to broadband excitation) frequencies combine. Additionally, the method of adjustments is not the most accurate method for measuring JND, but was selected for its relative speed [4], since the task of tracking the multi-frequency signals used in this experiment was not trivial and limiting participants' fatigue was a major concern. A similar protocol employing a more accurate method of estimating JND would give additional insight into precisely how the perceptual thresholds at two different frequencies integrate in motions combining both motion frequencies.

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