# Supplemental Material: "Dimer-dimer collisions at finite energies in two-component Fermi gases"

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In this supplemental material we provide additional details of our model for dimer-dimer relaxation in terms of the transition probability obtained from our numerical calculations.

The key observation in our model is that the inelastic transitions leading to deeply bound molecular final states can only occur when *at least* three atoms are enclosed at distances comparable to $r_0$. In the hyperspherical representation, the decay pathway in which only three atoms participate is viewed as an infinite series of avoid crossing between the initial dimer-dimer channel and all possible final states, as illustrated in Fig. 1(a), where the red solid curve describe the initial collision channel and the green dashed curves some of the possible final states.

Therefore, for a given $a$, the inelastic transition to a particular final state can be described in terms of the Fermi Golden rule

$$T_p^{(\lambda)}(a) \propto \left|\langle \Psi_{dd}\left(R,\Omega\right)\left|V\left(R,\Omega\right)\right|\Psi_\lambda\left(R,\Omega\right)\rangle\right|^2 \qquad (1)$$

where $\Psi_\lambda$ is the final state wave function, labeled by the quantum number $\lambda$, $\Psi_{dd}$ is our fully coupled dimer-dimer wavefunction, and $V$ is the sum of the interatomic interactions. The hyperangular behavior of the integrand is assumed to be proportional to the probability amplitude of three particles being in close proximity. On the other hand, the hyperradial behavior of the outgoing channel will oscillate very quickly away from the classical turning point, as shown schematically in Fig. 1(b). The fast oscillation will, in general, cancel out in regions away from the classical turning point, $R_\lambda$. This indicates that the integral over the hyperradius will be proportional to the area within the first oscillation of $\Psi_\lambda(R)$ times the remaining hyperradial behavior evaluated at $R_\lambda$. Based on these considerations, the squared result yields

$$T_p^{(\lambda)}(a, R_\lambda) \propto \frac{1}{R_\lambda}\left|F_{dd}\left(R_\lambda\right)\right|^2 \mathcal{F}\left(R_\lambda\right), \qquad (2)$$

where $F_{dd}$ is the dimer-dimer hyperradial wave function and $\mathcal{F}\left(R_\lambda\right)$ is the probability of having three out of four atoms at distances comparable to $r_0$ at hyperradius $R_\lambda$. Therefore, the inelastic transitions to a particular final state $\lambda$ occurs in the vicinity of $R_\lambda$ and it is governed by the probability of having the three atoms within distances comparable to $r_0$.

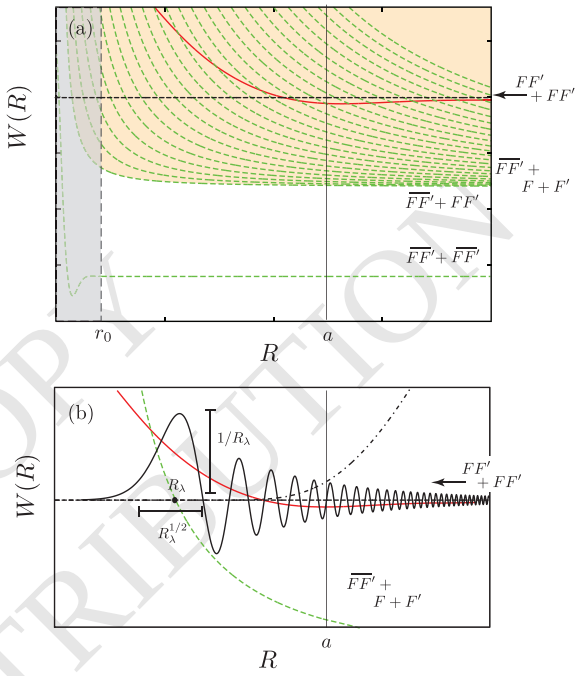


FIG. 1: (color online). (a) A schematic representation of the dimer-dimer relaxation process is shown. The red-solid solid curve represents the effective potential for the initial dimer-dimer channel, $FF' + FF'$, and green-dashed curves represent the possible final decay channels involving at least one deeply bound molecular state $\overline{FF}'$. Inelastic transitions to this almost continuum of final states are allowed for all values of $R$ (see main text). (b) Qualitative representations of the behavior of the hyperradial solutions in the dimer-dimer channel (black dot-dashed curve) and in a particular final channel (solid black curve) are shown to illustrate that the inelastic transition is more likely near the classical turning point $R_\lambda$.

In practice, we have calculate $\mathcal{F}(R_\lambda)$ by defining the proximity operator

$$f(R,\Omega) = e^{-(r_{12}^2+r_{23}^2)/2r_0^2} + \text{ cyclic permutations}, \qquad (3)$$

which is non-zero only when three atoms are sufficiently close to each other. $\mathcal{F}$ is then simply defined as

$$\mathcal{F}(R) \propto \langle \Phi_{dd}(R;\Omega)|f(R,\Omega)|\Phi_{dd}(R;\Omega)\rangle, \qquad (4)$$

where $\Phi_{dd}$ is our fully coupled dimer-dimer channel function and the integration is taken over all the hyperangles.

Our model for dimer-dimer relaxation, therefore, is simply obtained by summing Eq. (2) over the near continuum of $\lambda$ states, approximated by an integral over the

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classical turning points, which yields

$$V_{\rm rel}^{dd} \propto \frac{1}{k_{dd}} \int T_p^{(\lambda)}(a, R_\lambda)\rho(R)dR$$
$$= \frac{1}{k_{dd}} \int \frac{1}{R_\lambda} |F_{dd}(R_\lambda)|^2 \mathcal{F}(R_\lambda)\rho(R_\lambda)dR_\lambda$$
$$\approx \frac{1}{k_{dd}} \int \frac{P_{\rm WKB}(R_\lambda)\mathcal{F}(R_\lambda)}{R_\lambda \kappa(R_\lambda)} \rho(R_\lambda)dR_\lambda, \qquad (5)$$

where $k_{dd}^2 = 2m(E + 2E_b)$, $\rho(R_\lambda)$ is the nearly constant density of states and $|F_{dd}(R_\lambda)|^2$ was approximated by the WKB wavefunction in the classically forbidden region, $|F_{dd}(R_\lambda)|^2 \approx P_{WKB}(R_\lambda)/\kappa(R_\lambda)$. Here, $P_{WKB}$ is the WKB tunneling probability for the dimer-dimer hyperradial wavefunction and $\kappa^{WKB}(R_\lambda)$ is the WKB wavenumber.

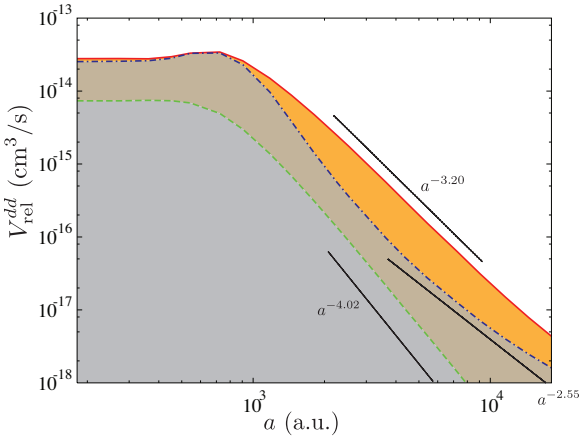


FIG. 2: (color online). The vibrational relaxation rate for different decay pathways is shown. The red-solid curve is the total rate given by Eq. (5), the green-dashed curve is the contribution from short-range inelastic transitions where all four-atoms participate in the collision. The blue-dot-dashed curve is the contribution from inelastic transitions near $R = a$ involving only three-atoms illustrating the effects due to the presence of the forth atom (see main text).

In our model for relaxation, breaking up the integration over $T_p(a)$ [Eq. (5)] allows us to analyze the contributions from different pathways. In Fig. 2 we show the total rate as a solid-red curve, obtained by integrating Eq. (5) from $2r_0$ to $10a$ [1]. In addition to that, we also plot the results for $V_{\rm rel}^{dd}$ obtained by integrating Eq. (5) from $2r_0$ to $5r_0$, see green-dashed curve in Fig. 2. This result determines the contribution from inelastic transitions which occur predominantly when all four atoms are within distances comparable to $r_0$. The blue-dot-dashed curve in Fig. 2, however, shows our results obtained by integrating from $a$ to $4a$, determining the behavior of the contributions from inelastic transitions that occurs near $R = a$. The main difference between this contribution and the total rate comes from the inelastic transitions for $R < a$. Although numerically we are unable to go to larger values of $a$, it is clear that the contributions for transitions near $R = a$ becomes increasingly more important and in the very large $a$ limit we expect these contributions to dominate the total rate. Interestingly, for the values of $a$ we studied, the contribution for transitions near $R = a$ already falls off slower than the $a^{-2.55}$ prediction of Ref. [2]. Therefore, we conclude that the mechanism that leads to the $a^{-2.55}$ suppression, although, qualitatively correct, doesn't quantitatively describe the transitions near $R = a$ due to the presence of the fourth atom.

[1] We have found that integrating out $T_p$ from $R = 2r_0$ up to $10a$ is enough to ensure that contributions for $R < 2r_0$ and $R > 10a$ are negligible.

[2] D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, Phys. Rev. Lett. **93**, 090404 (2004); Phys. Rev. A **71**, 012708 (2005).