

Introduction to Iterative Modeling Activity with Sample Explanation

A 1.0 kg mass is connected to a spring of spring constant 0.25 N/m. If the spring is displaced 0.50 m from equilibrium, how far does it travel in 1 second? Assume that this is not enough time for it to reach its equilibrium position

Relevant Equations:

$$F = -kx$$

$$a = F/m$$

$$v = v_0 + at$$

$$x = x_0 + vt$$

Let's choose 0.25 seconds to iterate over.

First Iteration: 0.00 - 0.25 seconds:

$$F = -kx = 0.25 \text{ N/m} \times 0.5 \text{ m} = -0.125 \text{ N}$$

$$a = F/m = 0.125 \text{ N} / 1 \text{ kg} = -0.125 \text{ m/s}^2$$

$$v = v_0 + at = 0 + (-0.125 \text{ m/s}^2)(0.25 \text{ s}) = -0.03125 \text{ m/s}$$

$$x = x_0 + vt = 0.5 \text{ m} + (-0.03125 \text{ m/s})(0.25 \text{ s}) = 0.4922 \text{ m}$$

Second Iteration: 0.25 - 0.50 seconds:

$$F = -kx = 0.25 \text{ N/m} \times 0.4922 \text{ m} = -0.1231 \text{ N}$$

$$a = F/m = 0.1231 \text{ N} / 1 \text{ kg} = -0.1231 \text{ m/s}^2$$

$$v = v_0 + at = -0.03125 \text{ m/s} + (-0.1231 \text{ m/s}^2)(0.25 \text{ s}) = -0.06203 \text{ m/s}$$

$$x = x_0 + vt = 0.4922 \text{ m} + (-0.06203 \text{ m/s})(0.25 \text{ s}) = 0.4767 \text{ m}$$

Third Iteration: 0.50 seconds - 0.75 seconds:

$$F = -kx = 0.25 \text{ N/m} \times 0.4767 \text{ m} = -0.1192 \text{ N}$$

$$a = F/m = 0.1192 \text{ N} / 1 \text{ kg} = -0.1192 \text{ m/s}^2$$

$$v = v_0 + at = -0.06203 \text{ m/s} + (-0.1192 \text{ m/s}^2)(0.25 \text{ s}) = -0.09183 \text{ m/s}$$

$$x = x_0 + vt = 0.4767 \text{ m} + (-0.09183 \text{ m/s})(0.25 \text{ s}) = 0.4537 \text{ m}$$

Fourth Iteration: 0.75 seconds - 1.00 seconds:

$$F = -kx = 0.25 \text{ N/m} \times 0.4537 \text{ m} = -0.1134 \text{ N}$$

$$a = F/m = 0.1134 \text{ N} / 1 \text{ kg} = -0.1134 \text{ m/s}^2$$

$$v = v_0 + at = -0.09183 \text{ m/s} + (-0.1134 \text{ m/s}^2)(0.25 \text{ s}) = -0.1202 \text{ m/s}$$

$$x = x_0 + vt = 0.4537 \text{ m} + (-0.1202 \text{ m/s})(0.25 \text{ s}) = 0.4237 \text{ m}$$

*Note that this is one way to do it, but there are others. Such as using $x = x_0 + \frac{1}{2} a(t - t_0)^2 + v_0(t - t_0)$ to update the position over each time interval.